

A Unit Plan on Probability & Statistics

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Education 352

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* The webquest I created, entitled “The Best of the Best: Probability & Basketball,” may be used appropriately between the two starred lesson plans. Instructions are provided within the webquest.

<http://users.manchester.edu/Student/JMFauser/WebQuest352/index.htm>

A. TEXTBOOK INFORMATION/COURSE INFORMATION

NAME OF COURSE/GRADE LEVEL:

Algebra 2/Eleventh Grade

DESCRIPTION OF COURSE:

This course is a continuation of Algebra I. It is assumed the student has knowledge of basic algebraic concepts. Functions and their applications will be discussed more in depth.

NAME OF CHAPTER/UNIT:

Probability & Statistics

DESCRIPTION OF CHAPTER/UNIT:

This chapter/unit is an introduction to probability and how it is used to generate statistics.

TITLE OF TEXTBOOK:

Algebra 2

NAMES OF AUTHORS/EDITORS:

Dr. Randall I. Charles, Dan Kennedy, Basia Hall, Allan E. Bellman, Sadie Chavis Bragg, William G. Handlin, Laurie E. Bass, Art Johnson, Stuart J. Murphy, and Grant Wiggins

NAME OF PUBLISHING COMPANY:

Pearson Education Inc.

COPYRIGHT DATE:

2011

READING LEVEL OF TEXTBOOK:

This textbook is designed for students who have already completed an Algebra I course. Basic operations and definitions are assumed, and are utilized in more complex problems.

B. PHILOSOPHY OF READING IN THE CONTENT

STANDARDS

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

A2.8.2 - Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

A2.8.3 - Know and apply the characteristics of the normal distribution.

- Identify settings in which the normal distribution may be useful.
- Determine whether a set of data appears to be uniform, skewed, or normally distributed.
- Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two, or three standard deviations about the mean.

A2.8.4 - Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

IMPORTANCE

Learning about probability and statistics prepares students for future required math courses they may take in college. Probability and statistics have many real world applications that may be required for careers in fields such as environmental sciences, medical research, government, and meteorology.

PHILOSOPHY

Reading for math is not like reading for other subjects. Often there is a short, concise paragraph explaining a mathematical idea followed by several examples. There is no storyline that draws the reader's attention. When there are stories in math text books, it may be difficult for the student to identify the pertinent points of the story. An understanding of common math symbols is also necessary for math reading comprehension. If students are not taught how to read math books, it will be difficult for them to succeed in higher level courses.

It is important to teach students how to read their math textbooks. Some lessons are easily taught by giving examples without using the book, and this is often the case. But when students are asked to learn a section on their own by reading, they experience some difficulty sorting through the information. I believe if students know what to look for when they read their books they will know how to approach assignments that require them to teach themselves. It is my goal to help students gain the skills needed to read and comprehend the material in their textbooks. Teaching students how to read math books will help students build confidence in their math abilities.

C. ANNOTATED LIST OF TRADE BOOKS FOR MATHEMATICS

Balliett, B. (2004). *Chasing Vermeer*. New York: Scholastic, Inc.

Petra and Calder are two sixth graders with a newfound friendship. Their love of art and their problem solving skills bring them to the rescue when a thief steals a valuable Vermeer painting. The duo discovers a series of clues to the whereabouts of the painting in the newspaper. Petra and Calder take it upon themselves to follow these clues and to recover the Vermeer. This novel tests the reader's logic and reasoning skills as they chase down the Vermeer along with Petra and Calder.

Balliett B. (2007). *The Wright 3*. New York: Scholastic, Inc.

Petra and Calder are at it again, along with Calder's friend Tommy. Their teacher presents them with a problem: Frank Lloyd Wright's Robie House is falling apart, and is going to be separated and sold to four different museums. This book combines geometry and codes with art and literature as the three join forces to save the historic house from its demise.

Bittinger, M. L. (2004). *One Man's Journey through Mathematics*. Pearson Education, Inc.

This book is exactly what the title indicates: it is the story of one man's journey through mathematics. Professor Marvin L. Bittinger walks us through how mathematics has had an impact on his life. He discusses a wide variety of mathematical applications from in the classroom to comics to sports, and even to hamburgers. There are topics in the book to which anyone could relate. Although Bittinger is the author of many math textbooks, this book is not written like a textbook. It is instead a collection of life stories and even contains some math problems for the math buffs out there. This book helps the reader gain an appreciation for math in our everyday lives. (Also, an interesting fact is that Bittinger completed his undergraduate studies at Manchester College.)

Ellis, J. (April 2004). *What's Your Angle, Pythagoras?* Charlesbridge Publishing, Inc.

This fictional book depicts the story of Pythagoras and how he developed his famous theorem. It tells of his travels with his father from Samos to Alexandria and his fascination with triangles, especially right triangles. Along his journey he meets a builder who encourages Pythagoras curiosity. This book provides a unique and clear explanation of the math behind the Pythagorean Theorem.

Pappas, T. (January 1993). *Math Talk: Mathematical Ideas in Poems for Two Voices*. San Carlos, California: World Wide Publishing/Tetra.

This book provides an interesting way to introduce new math concepts. Mathematical ideas are presented in the form a poem. This book may not be for all students, but it does appeal to linguistic and musical learners. It does not go into much depth, but it can be used as a tool to help remember a variety of concepts.

—

I can use trade books in my classroom in a variety of ways to enhance how I teach math to my students. Depending on the book, I can use a portion of it as a “hook” to get students excited about what they are about to learn. Also, I can use trade books to connect with different learning styles. Reading about a concept may be more appealing to some students rather than physical demonstrations. Trade books can also be used to show students the numerous real world applications that mathematics has to offer.

D. LESSON PLAN TO ACTIVATE PRIOR KNOWLEDGE

Lesson: Introduction to Unit/Probability & Statistics

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

Performance Objectives:

Students will create a graph with 100 percent accuracy when given a data set.

Assessment:

Students will work together in pairs to create graph that organizes the data collected during the lesson. They will be graded on complete and correct work. It will be turned in at the end of class.

Advanced Preparation by Teacher:

Materials -

- Dry erase board
- Dry erase markers
- Graph paper
- Stopwatch
- Colored pencils or markers
- Tape

Preparation -

Make sure there are enough copies of the “Percentage Race” worksheet and graph paper for each student.

Procedure:

Introduction to Unit -

Think-Pair-Share: To introduce this unit, Probability & Statistics, have the students get out a half sheet of paper. Ask the students how they think probability and statistics are used daily (Bloom: Level III - Application). Have them make a list of at least three ideas. After two minutes have them pair up with the person next to them, and ask them to share their ideas with one another (Gardner: Interpersonal). This should only take about a minute. Then have the students return to their seats, and as a class share what ideas they found. After a brief discussion, explain to the students that this unit will cover a variety of ways that probability and statistics are used in everyday life. Collect their idea sheets because they may be used later to help create some example/homework problems later in the unit.

Plan -

*This lesson will begin with review of how to calculate percentages and how to create graphs out of given data.

1. Pass out the “Percentage Race” worksheets out to each student face down (Bloom: Level I - Knowledge).
 - They will be racing to finish the worksheet in the fastest time.
 - These questions should all be review for the students.
 - Explain that they will be timed and that once they finish, they will need to stand up and write down the time that was called out.
2. Once everyone has the worksheet, say “go” and start the stop watch.
 - As the students finish, have them stand up (Gardner: Logical-Mathematical and Kinesthetic). When they stand up, call out the time on the stop watch. The students will write this down.
 - After everyone has finished, have them sit back down.
3. Go over the answers with the students using the answer key that is attached.
 - Have the students mark and correct any questions they may have gotten wrong (Gardner: Intrapersonal). They should get a score out of 5.
 - Ask them to be honest, because this data will be used a little later in the lesson.
 - Remind them that this activity is just for fun and is just a review.
 - They will not be graded on how they did on the worksheet.
4. Have each student go to the board and write their time and score for the worksheet (Gardner: Visual). Explain that they will use this data to create graphs.
5. Ask the students to pair back up with their partner from earlier (Gardner: Interpersonal). Tell them that they will be working together to create a graph representing the information on the board (Gardner: Visual).
6. Pass out the graph paper.
7. Help the students set their graph paper up with the correct labeling scales on the axes. See attached sheet.
8. Explain to the students that they will be creating a bar graph with the times recorded. They will have a total of 6 bars (0-1 minute, 1-2 minutes, 2-3 minutes, 3-4 minutes, 4-5 minutes, and 5+ minutes). On the same graph, they will create a line plot of scores for each bar and for each possible score. There will be a total of 6 line plots (scores of 0, 1, 2, 3, 4, 5). See the attached graph paper. The students may use the colored

pencils/markers to make their graphs.

9. Walk around and assists students that need help with this activity.

10. As they finish, have the students discuss the results in their pairs as they wait for everyone to finish (Bloom: Level II - Comprehension).

Closure –

Once everyone has finished, have the students put their names on their graphs and tape them on the board at the front of the room. They should see that the graphs are identical. Ask the students if they have any questions about graphing. Answer any questions they might have. Now have them return to their seats. For homework, ask them to write a paragraph about why we graph data. Some questions they might consider are: How do we use graphs? Why are there different kinds of graphs? Who uses graphs? (Bloom: Level II - Comprehension, Level III - Application). Let them know that tomorrow they will be learning the vocabulary for the unit.

Adaptations/Enrichment:

Learning disabilities in reading comprehension - The problems on the worksheet can be read aloud, and oral instructions can be given.

Self-Reflection:

Was the review too easy or not in depth enough? Was there enough time allotted for this lesson? Were the questions on the “Percentage Race” worksheet too easy? Did the homework question effectively get the students to think about why we use graph data?

Pre-Test: The following document is a Pre-Test that is to be used in the class before this lesson. It is not intended to be taken for a grade. The students will take the Pre-Test prior to this lesson and turn it in. The results of this pre-test will help guide instruction throughout the lesson.

Name: _____

Score: ____/8

Unit Pre-Test: Probability & Statistics

Directions: Answer each question as best as you can. You may use a calculator. (1 point each)

Evaluate the following:

1. $5!$
2. ${}_{10}P_4$
3. ${}_8C_3$
4. What is the probability of rolling a 3 when you roll a standard dice.
5. Find the mean of: 2 4 4 7 9 2 2
6. What is the mode of the data set in #5?
7. What is the median of #5?
8. What is standard deviation?

KEY

Unit Pre-Test: Probability & Statistics

- 1. 120**
- 2. 5040**
- 3. 53**
- 4. $\frac{1}{6}$**
- 5. 4.3**
- 6. 2**
- 7. 7**
- 8. Measure showing how much data values deviate from the mean.**

Name: _____ Time: _____

Percentage Race

Directions: Race to find the percentages. Stand up when you are finished and listen for your time. Write your time in the given space. You may **NOT** use a calculator.

1. What is 13 percent of 33?

2. What is 64 percent of 80?

3. What percent is 45 out of 72?

4. What percent is 3 out of 5?

5. If a t-shirt costs \$12.50 and you have a coupon for 30% off, how much will the shirt be if you use the coupon?

Stand up!!

Name: _____ Time: _____

KEY

Percentage Race

Directions: Race to find the percentages. Stand up when you are finished and listen for your time. Write your time in the given space. You may **NOT** use a calculator.

1. What is 13 percent of 33?

4.29

2. What is 64 percent of 80?

51.2

3. What percent is 45 out of 72?

62.5%

4. What percent is 3 out of 5?

60%

5. If a t-shirt costs \$12.50 and you have a coupon for 30% off, how much will the shirt be if you use the coupon?

\$8.75

Stand up!!

E. LESSON PLAN TO INTRODUCE NEW VOCABULARY

Lesson: Unit Vocabulary

Length: 50 minutes

Age/Grade Intended: Algebra 2

Academic Standard(s):

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

A2.8.3 - Know and apply the characteristics of the normal distribution.

A2.8.4 - Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Performance Objectives:

Students will define the unit vocabulary when given a Math Journal form with complete accuracy.

Assessment:

Students will be shown how to correctly organize their Math Journals (see attached sheets and procedures). They will be asked to fill out their journals as new words are introduced. Students will be graded based on completion and accuracy. Their journals will be checked at the beginning of class the each day. The journals will be turned in at the end of the unit.

Advanced Preparation by Teacher:

Materials -

Dry erase board

Dry erase markers

Preparation -

No preparation is needed.

Procedure:

When the bell rings, collect the paragraph that is due today. Ask the students if they have any lingering questions about yesterday's introduction lesson. If they have any questions, answer them, and then begin today's lesson.

Introduction -

Have the students arrange the desks into a circle for a class discussion. Ask them the following questions allowing about 2-3 minutes of discussion time for each question.

- Why do we use graphs with data? What are the advantages? Disadvantages?
- How does graphing relate to probability and statistics?
- Can you think of any careers that would use the graphing of probabilities and/or statistics?

As the students share their answers, write their ideas on the board and help guide the

discussion.

At the end of the discussion, explain to the students that they will be learning more about how probability, statistics, and graphing are used every day in different careers.

Tell the students that today they will be learning the unit vocabulary. Explain to the students that learning the vocabulary will be an important part to this unit. Knowing how to use the vocabulary will help them find applications in the real world. Today they will just be introduced to the terms, and later in the unit, they will learn how to use them.

Plan -

1. Explain to the students that they will be using **Math Journals** to organize the unit vocabulary. Journals can be used in other subjects to organize important information as well.

2. Show the students, on the board, the format they will be using for their journals:

Def.

Eqn.

Pic.

Ex.

Uses:

3. Have the students get out three sheets of loose leaf paper and fold them in half (top to bottom).

4. Tell the students to copy the format onto the first half of one of the sheets of paper. Make sure they leave enough space in between each category. They will use the same format for each word. Tell the students to fill out their journals as each new word is introduced.

***Note:** Not all categories will be filled out today. The journals will be used throughout the unit.

5. On the board, write down the first vocabulary word: **permutation**.

- Read aloud the definition: A **permutation** is an arrangement of items in a particular order.

- Write on the board: order matters, next to permutation.

- Explain that permutations are a way to count an arrangement of items when order matters.

- Next to Eqn, write the following equation: $nPr = n!/(n-r)!$

- Explain that tomorrow they will learn more about how to use permutations.

***Note:** No Pic, Ex, or Uses for this term yet.

6. Next, write: **combination**
 - Read aloud the definition: A **combination** is a selection in which order does not matter.
 - Write on the board: order does NOT matter, next to combination.
 - Next to Eqn, write: $nCr = n!/r!(n-r)!$
 - Tomorrow they will also learn about combinations.
 - ***Note:** No Pic, Ex, or Uses yet for this term.

7. Write: **probability**
 - Read the definition: **Probability** measures how likely it is for an event to occur.
 - Write the definition on the board.
 - Next to Eqn, write: $P(\text{event}) = \# \text{ times event occurred} / \text{total \#}$
 - ***Note:** No Pic, Ex, or Uses yet for this term.

8. Write: **sample space**
 - Read the definition: The set of all possible outcomes.
 - Write the definition on the board.
 - ***Note:** No Eqn, Pic, Ex, or Uses yet for this term.

9. Write: **equally likely**
 - Read the definition: If each outcome has the same chance of occurring, then the events are **equally likely**.
 - Write: events have the same chance of occurring.
 - ***Note:** No Eqn, Pic, Ex, or Uses yet for this term.

10. Write: **dependent events**
 - Read the definition: The occurrence of one event affects the probability of the second event.
 - Write the definition on the board.
 - ***Note:** No Eqn, Pic, Ex, or Uses yet for this term.

11. Write: **independent events**
 - Read and write the definition on the board: The outcome of one event does not affect the probability of another event.
 - Next to Eqn, write: $P(A \text{ and } B) = P(A) \cdot P(B)$
 - ***Note:** No Pic, Ex, or Uses yet for this term.

12. Write: **mutually exclusive events**
 - Read and write the definition on the board: Two events that cannot happen at the same time.
 - Next to Eqn, write: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
 - ***Note:** No Pic, Ex, or Uses yet for this term.

13. Write: **conditional probability**
 - Read the definition: The probability that an event, B, will occur given that another event, A, has already occurred is called a **conditional**

probability.

- Next to Eqn, write: $P(B|A) = P(A \text{ and } B)/P(A)$

***Note:** No Pic, Ex, or Uses yet for this term.

14. Write: **Box-and-Whisker Plot**

- Read the definition: A **Box-and-Whisker Plot** is a way to display data that uses quartiles to bound the center box and the minimum and maximum values to form the whiskers.

- Write: A way to display data.

***Note:** No Eqn, Pic, Ex, or Uses yet for this term.

15. Write: **variation**

- Read the definition: Describes how the data in a data set are spread out.

- Next to Eqn, write: $\sigma^2 = \Sigma(x-\mu)^2/n$

***Note:** No Pic, Ex, or Uses yet for this term.

16. Write: **standard deviation**

- Read the definition: The measure of the spread. It is the square root of the variation.

- Next to Eqn, write: $\sigma = \sqrt{\Sigma(x-\mu)^2/n}$

***Note:** No Pic, Ex, or Uses yet for this term.

17. Explain to the students that they will learn more about each vocabulary word in the lessons to follow. As they learn more about each word, they will be expected to fill out their Math Journals accordingly (Bloom: Level I - Knowledge, Level II - Comprehension). At this time, ask if they have any questions about their Math Journals.

18. Ask the students other ways they can use journals to help them learn. (Bloom: Level III - Application).

Closure -

Allow the students to get into groups of two or three (Gardner: Interpersonal). Using their book and Math Journals, have them make predictions in the "Uses" category for each word (Bloom: Level V - Analysis). Let them know that these should be educated guesses. They should work together until the bell rings. Walk around to answer any questions they may have.

Adaptations/Enrichment:

This lesson can be modified so that the beginning activity is done individually.

Self-Reflection:

Did I allow enough time for this lesson? Were the students able to find enough triangles in the activity? Did the activity show the connection between the material and real life? Was enough time spent on definitions? Will the journals help the students remember the vocabulary?

F. LESSON PLAN MODIFIED FOR ADD

Lesson: Permutations and Combinations

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.4

Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Performance Objectives:

Students will solve word problems using permutations, combinations, and other counting techniques when given a worksheet with 85 percent accuracy.

Assessment:

Students will be given a worksheet with 15 counting problems. They will be grade on the accuracy of their work. The worksheet is due at the beginning of class the next day.

Advanced Preparation by Teacher:

Materials -

Lunch menu for today

The Sundae Scoop by Stuart J. Murphy

Dry erase board

Dry erase markers

Preparation -

A copy of the children's book *The Sundae Scoop* by Stuart J. Murphy, and today's lunch menu from the cafeteria will need to be acquired before the lesson. Also, make sure there are enough copies of the "Count Me If You Can!" worksheet for each student.

Procedure:

Introduction -

Begin the lesson by explaining to the students that they will be learning different counting techniques such are permutations and combinations. **Read aloud to the class *The Sundae Scoop*.** It is a children's book that illustrates the mathematical concept of combinations.

After reading the story, ask the students what is a combination. (Bloom: Level I - Knowledge) Then ask them how they think combinations apply to probability and statistics. (Bloom: Level III - Application) Explain that combinations are a way of counting possible events. They will now learn two other counting techniques.

Plan -

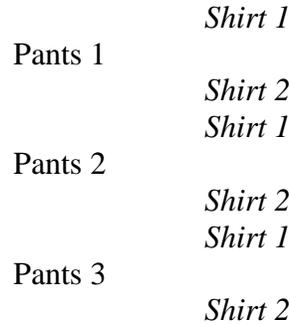
1. Ask the students to get out their Math Journals. They will be adding to the **permutation** and **combination** entries. They may also want a notebook to take additional notes.

2. On the board, write: Fundamental Counting Principle (FCP)
 - Explain that the FCP is a way to count using multiplication.
 - If event M can occur in m ways followed by event N that can occur in n ways, the event M followed by N can occur in $m \times n$ ways.

3. Examples for FCP: (These should also be written on the board.)

Example 1 - How many different outfits can you wear if you have 3 pants and 2 shirts to choose from.

- One way to solve is to make a tree diagram:



*There are 6 outfits to choose from

- Or you can use FCP: $3 \times 2 = 6$ outfits.

Example 2 - Using today's lunch menu, determine how many different meals there are to choose from by multiplying the number of sandwiches, sides, desserts, and drink options.

3. On the board write: Permutations - order matters.
 - Explain that permutations count the number of ways to order r objects selected from a set of n objects.
 - The equation is $nPr = \frac{n!}{(n-r)!}$
 - Explain that $n!$ means "n-factorial" and that $n! = n(n-1)(n-2)\dots 1$

4. Examples for permutations:

Example 1 - In how many different ways can you file 12 folders, one after another, in a drawer?

- Use FCP to count # of permutations

$$12 \times 11 \times \dots \times 1 = 12! = 479,001,600$$

- There are 479,001,600 ways to file 12 folders in a drawer.

Example 2 - Ten students are in a race. First, second, and third places will win medals. In how many ways can 10 runners finish first, second, and third.

- One way to solve is to use the FCP:

$$10 \times 9 \times 8 = 720$$

- Or you can use the formula for nPr :

$$10P3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$$

5. On the board, write: Combinations - treat the same.
- Explain that a combination is the number of ways you can choose r objects from a group of n objects.
 - The equation is $nCr = \frac{n!}{r!(n-r)!}$

6. Examples for combinations:

Example 1 - What is the number of combinations of 13 items taken 4 at a time?

- Use the formula to solve.

$${}_{13}C_4 = \frac{13!}{4!(13-4)!} = \frac{13!}{4!9!} = 715$$

7. Deciding whether to use permutations or combinations.

- You have to ask does order matter? If it does, use permutations. If order does not matter, use combinations.

- Read the following scenarios to the students and have them tell if you should use permutations or combinations (Bloom: Level II - Comprehension):

Scenario A - A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawing to display. In how many different ways can he select the drawings?

* Combination

Scenario B - You will draw winners from a total of 25 tickets in a raffle. The first ticket wins \$100. The second ticket wins \$50. The third ticket wins \$10. In how many different ways can you draw the three winning tickets?

*Permutation

Closure –

Have the students get into groups of three to create their own permutation and combination scenarios (Bloom: Level V - Synthesis)(Gardner: Interpersonal).

- They should create one permutation scenario and one combination.
- Have the groups write down their ideas.
- Walk around to help the groups that need it.

Once the groups have finished, allow them to share their scenarios with the class.

After they have finished sharing their scenarios, pass out the homework worksheet titled “Count Me If You Can!” (Gardner: Logical-Mathematical) They may work for the remaining time. Walk around to help any student who may need it. Also, be sure to remind them that their entries for permutations and combinations should be filled out in their Math Journals.

Adaptations/Enrichment:

ADD - For students with ADD the lecture time may not be engaging enough. This lesson can be modified to better engage students with this disability in the following way. Physical manipulatives or online manipulatives can be used during the examples. For Permutations Example 2, the ADD student along with 9 other classmates can be used to demonstrate the example (Gardner: Kinesthetic and Visual). This will help break up the lesson to help keep the ADD student focused.

Self-Reflection:

Were there enough examples? Did I pick good examples to illustrate each concept? Was there enough time for this lesson? Was the read aloud book engaging? Do the students understand the difference between permutation and combination?

Name: _____

Count Me If You Can!

Solve use the correct formulas.

1. $12!/6!$

2. $8P1$

3. $9P6$

4. $4C3$

5. $7C7$

6. $7C4/9C4$

7. Use the definition of permutation to show why $0!$ should equal 1.

Tell whether it is a *permutation* or a *combination*. Do not solve.

8. How many different teams of 11 players can be chosen from a soccer team of 16?

9. Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?

10. A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?

Solve.

11. Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes one student. In how many ways can ten time slots be assigned?

12. There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?

13. In how many different ways can you arrange nine CDs one after another on a shelf?

14. An old web-site requires a four-character password consisting of three numbers and one letter. A new website requires a six-character password consisting of three numbers and three letters. How many more passwords can be made for the new website?

15. A consumer magazine rates televisions by identifying two levels of price, five levels of repair frequency, three levels of features, and two levels of picture quality. How many different ratings are possible?

Name: _____

Count Me If You Can!

KEY

Solve use the correct formulas.

1. $12!/6!$

665,280

2. $8P1$

8

3. $9P6$

60,480

4. $4C3$

336

5. $7C7$

1

6. $7C4/9C4$

5/18

7. Use the definition of permutation to show why $0!$ should equal 1.

$nP0 = n!(n-0)! = n!/n! = 1$

Tell whether it is a *permutation* or a *combination*. Do not solve.

8. How many different teams of 11 players can be chosen from a soccer team of 16?

Combination

9. Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?

Combination

10. A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?

Combination

Solve.

11. Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes one student. In how many ways can ten time slots be assigned?

10,897,286,400

12. There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?

56

13. In how many different ways can you arrange nine CDs one after another on a shelf?

362,880

14. An old web-site requires a four-character password consisting of three numbers and one letter. A new website requires a six-character password consisting of three numbers and three letters. How many more passwords can be made for the new website?

17,500,000

15. A consumer magazine rates televisions by identifying two levels of price, five levels of repair frequency, three levels of features, and two levels of picture quality. How many different ratings are possible?

60

G. LESSON PLAN MODIFIED FOR LEARNING DISABILITIES

Lesson: Probability

Length: 45 minutes

Age/Grade Intended: Algebra 2

Academic Standard(s):

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

Performance Objectives:

Students will use relative frequency to estimate the probability of an outcome with 80 percent accuracy when assigned homework (21 problems) from the textbook.

Assessment:

Students will be assigned homework from the textbook: **p. 685-686 #1, 2, 5, 7, 8, 9, 15-23, 25, 26, 29, 32, 33, 34**. This homework will be due the next day at the beginning of class. The will be graded on work shown, how accurately they set up their probabilities, and correctness.

Advanced Preparation by Teacher:

Materials -

- Compass
- Dry erase markers
- Small, soft ball
- Masking tape
- Tape measure

Preparation -

Before the students arrive, draw a target with three circles on the board using the compass. The outermost ring should have a radius of one foot. The middle ring should have a radius of 6 inches. The innermost bull's-eye ring should have a radius of 3 inches. Measure a distance of ten feet from the board and mark this on the floor with a piece of masking tape.

Procedure:

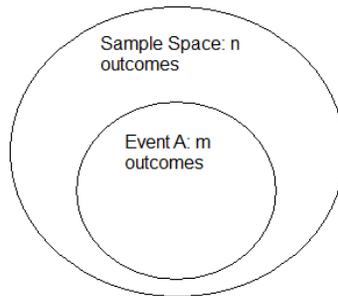
Introduction -

To demonstrate probability the students will use the target constructed before class to determine the probability that they hit a bull's-eye. Once the bell rings ask for a volunteer. This student will help mark where the ball hit's the target. Have the students take turns throwing the ball at the target (Gardner: Kinesthetic). If they do not hit the target, have them throw again. After each student is done, tally up the totals for the outermost ring, middle ring, and bull's-eye on the board. Have the students look at this data and have them speculate what the probability of hitting a bull's-eye is (Gardner: Visual) (Bloom: Level V - Synthesis). They will answer this question later in the lesson.

Plan -

1. Have the students return to their seats, and ask them get out their Math Journals for notes.
2. Write today's objective on the board: Estimate probability of an outcome using relative frequency.
3. Write: **probability**.
 - Def.: Measures how likely it is for an event to occur.
 - Explain to the students that there are three types of probability that they will learn today: Experimental, Theoretical, and Geometric. These should go under the Ex. section in their notes.
 - Ask them to recall the equation for probability (Bloom: Level I - Knowledge.)
$$P(\text{event}) = \frac{\# \text{ times event occurred}}{\text{total}\#}$$
4. Write: **experimental probability**.
 - Tell the students that when you gather data from observations you can calculate **experimental probability**. Each observation is an experiment or a trial.
 - Eqn.: $P(\text{event}) = \frac{\# \text{ times event occurs}}{\# \text{ trials}}$.
 - Give an example of experimental probability: A softball player got a hit in 20 of her last 50 times at bat. What is the experimental probability that she will get a hit in her next at bat?
 - * Ask the students what is the trial? The event? (Bloom: Level II - Comprehension). The trial is an at bat and the event is a hit.
 - * $P(\text{hit}) = \frac{\# \text{ hits}}{\# \text{ at bats}} \rightarrow \frac{20}{50} = .4$
 - Data is being gathered from actual trials or a simulation.
5. Explain to the students that sometimes it is difficult or unreasonable to conduct trials. Ask the students to think of such a situation (Bloom: Level III - Application). Simulations are used to estimate experimental probability. The more times you run a simulation the more accurate the average becomes.
6. Write: **theoretical probability**.
 - Def.: If a sample space has n **equally likely** outcomes and an event A occurs in m of these outcomes, then the **theoretical probability** of event A is $P(A) = m/n$.
 - Eqn.: $P(A) = m/n$

- Pic.:



7. Write: **sample space**.

- Def.: the set of all possible outcomes to an experiment.

8. Write: **equally likely**.

- Def.: each outcome in the sample space has the same chance of occurring.

9. Show an example of theoretical probability: What is the theoretical probability of each event?

i) getting a 5 on one roll of a fair die.

$$- P(5) = 1/6$$

ii) getting a sum of 5 on one roll using two dice.

$$- P(\text{sum of } 5) = 4/36 = 1/9$$

10. Write: **geometric probability**.

- Def.: using areas to find theoretical probability

- Example: A batter's strike zone depends on the height and stance of the batter. What is the geometric probability that a baseball thrown at random within the batter's strike zone, as shown in the text book, page 684, will be a high-inside strike.

- Draw the picture in the book on board.

- $P(\text{high-inside strike}) = \text{area of high-inside strike zone} / \text{area of total strike zone}$.

$$= 4 \cdot 6 / 17 \cdot 22 = .064$$

Closure -

Ask the students what kind of probability the target activity at the beginning of the lesson was (Bloom: Level II - Comprehension). (Geometric probability) Ask the students to create an equation to find the probability of hitting a bull's-eye given that the radius of the out ring is 1 foot, the radius of the middle ring is 6 inches, the radius of the bull's-eye is 3 inches, and the area formula is: $A = \pi r^2$ (Bloom: Level III - Application). They should get that $P(\text{bull's-eye}) = \text{area of bull's-eye} / \text{total area} = 28.27 / 452.39 = .062$ (Gardner: Mathematical-Logical). Does this fit the data collected from the class? Use what was marked on the board. $P(\text{bull's-eye}) = \# \text{ hits in center} / \text{total hits}$. Have the

students compare these probabilities (Bloom: Level VI - Evaluation). Assign the students the homework problems from the book: **p. 685-686 #1, 2, 5, 7, 8, 9, 15-23, 25, 26, 29, 32, 33, 34**. They may work on the assignment until the bell rings. Walk around to see if they have any questions.

Adaptations/Enrichment:

Learning Disability - This lesson plan can be modified in a variety of ways for students with learning disabilities. I can have copy or an outline of my notes ready for the student before class. For the homework, I can create a problem set that has fewer problems, but still covers what he/she is expected to know. Also, someone could read the problems for them if it is easier to process.

Self-Reflection:

Did the students enjoy the target activity? Were they able to relate it to the lesson? Were the definitions clear? How effective were the examples I chose? Did I have enough time for everything?

H. LESSON PLAN MODIFIED FOR GIFTED AND TALENTED

Lesson: Probability of Multiple Events

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.2 – Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

Performance Objectives:

Students will determine the probability of simple events involving independent and dependent events with 85% accuracy when given a worksheet containing 25 problems.

Assessment:

Students will be given a worksheet with 25 problems. They will be graded on work shown, how accurately they set up their probabilities, and correctness. It will be possible to receive partial credit for some problems. Point values for each problem are given on the worksheet. The worksheet will be due at the beginning of class the next day.

Advanced Preparation by Teacher:

Two standard decks of 52 playing cards

Probability of Multiple Events Worksheet

Procedure:

- *Intro:* Use the decks of cards to demonstrate the difference in probabilities of single events (from previous lesson) and probabilities of multiple events.
 - Single events:
 - $P(\heartsuit)?$ (1/4)
 - $P(\text{Jack})?$ (1/13)
 - $P(5\clubsuit)?$ (1/52)
 - Make conjectures about the following events:
 - $P(3\heartsuit \text{ then } Q\spadesuit \text{ from the same deck})?$
 - $P(A\diamond \text{ from first deck \& } a\spadesuit \text{ from the second deck})?$
 - $P(8 \text{ \& } K)?$
 - $P(7 \text{ or Red})?$
 - Explain that these are examples of multiple events because the probability of more than one event is being taken into account. In other words, more than one thing is happening.

- To find the probability of two events occurring together, you have to decide whether one event occurring affects the other.
- Have the students get out their Math Journals and note taking materials.
- Classifying Events
 - **Dependent events**
 - Def.: The occurrence of one event affects how a second event can occur.
 - Eqn.: None
 - Pic.: None
 - Ex.: Pick one flash card, then another from a stack of 30 flash cards.
 - **Independent events**
 - Def.: The occurrence of one event does not affect the probability of another event. **AND**, \times
 - Ex.: Roll a number cube. Then spin a spinner.
 - Pic.: None
 - Eqn.: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Finding the Probability of Independent Events
 - **Example** – In the cafeteria, there are 10 chocolate milk cartons and 5 regular mild cartons. There are also 8 packages of chocolate chip cookies and 12 packages of peanut butter cookies. If you grab a milk and package of cookies without looking, what is the probability you get regular milk and chocolate chip cookies?
 - Event A = picking regular milk; Event B = picking chocolate chip cookies
 - A and B are independent.
 - $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{5}{15} + \frac{8}{20} = \frac{11}{15}$
 - $P \approx 73\%$
- Mutually Exclusive Events and Finding Probability
 - **Mutually exclusive events**
 - Def.: Two events that cannot happen at the same time. **OR**, $+$
 - Ex.: When rolling a dice, you roll a 2 and a 3.
 - Pic. Venn Diagrams can be useful when finding the probability of mutually exclusive events.
 - Eqn.: $P(A \text{ or } B) = P(A) + P(B)$
 - (If A and B are independent, then $P(A \text{ or } B) = P(A) + P(B)$)
 - **Example** – Students at a high school can take one foreign language each term. About 37% take Spanish, and about 15% take French. What is the probability that a student chosen at random is taking Spanish or French?
 - $P(\text{Spanish or French}) = P(\text{Spanish}) + P(\text{French})$
 - $.37 + .15 = .52$
- Finding Probability
 - If two events are not mutually exclusive: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

- **Example** - The numbers 1 through 10 are written on index cards and placed in a box. What is the probability that a card chosen at random has a number greater than 7 or even?
 - $P(>7 \text{ or even}) = P(>7) + P(\text{even}) - P(>7 \text{ and even})$
 - $\frac{3}{10} + \frac{5}{10} - \frac{2}{10} = .60$
- Go back to the Intro to check the predictions made at the beginning of the lesson.
 - P(drawing one card then Q♠ from the same deck)?
 - Dependent Events – depends on the first draw
 - P(A♦ from first deck & a ♠ from the second deck)?
 - Independent Events: $\left(\frac{1}{52}\right) \cdot \left(\frac{1}{4}\right) = \frac{1}{208}$
 - P(8 or K)?
 - Mutually Exclusive Events: $\frac{1}{13} + \frac{1}{13} = \frac{2}{13}$
 - P(7 or Red)
 - Not mutually exclusive: $\frac{1}{13} + \frac{1}{2} - \frac{2}{13} = \frac{11}{26}$

Adaptations:

- Instead of giving the equations for finding independent event probability and mutually exclusive probability, start by providing examples, and then have the students construct the equations.
- Have the students create a “How To” plan for finding the probabilities of each kind of events.
- Create a worksheet with more challenging problems.

It is crucial for teachers to know their students as learners. Not only is it important to take into consideration the different needs students with disabilities may have, it is also important to consider the needs of any gifted and talented students you may have when writing lesson and unit plans. Just because these students might understand the material you are teaching the class does not mean that we are meeting their needs as learners. These students still need to be engaged and challenged by the material.

In this particular unit on probability and statistics, there are a variety of enrichment activities and ways to differentiate the material to benefit the needs of gifted and talented students. For example, when assigning homework, especially from the first few sections that deal with simpler concepts such as probability of simple events and counting techniques, I can choose questions that require a higher order of thinking. I could challenge them by asking them why things, such as the equations for permutations and combinations, work. I could also have them derive these formulas. Since basic computing problems might bore these types of learners, it would be more beneficial to limit these computing problems and provide them with more challenging questions like the ones previously mentioned.

There are numerous ways to show students how to apply probability and statistics to real world careers and problems. One type of enrichment activity that would go along with probability is showing students how probability is used in other fields. More specifically, I could show them how a biologist uses Punnett Squares and probability to predict possible gene combinations. When we discuss statistics, I could have them conduct their own survey or experiment and analyze the data they collect by choosing the appropriate methods described in class. Having them analyze their data and methods they chose to use would challenge the students to find meaning in their results.

Name: _____

Probability of Multiple Events

#1-4 Classify each pair of events as *dependent* or *independent*. (1pt each)

1. A month is selected at random; a number from 1 to 30 is selected at random.
2. A month is selected at random; a day of that month is selected at random.
3. A letter of the alphabet is selected at random; one of the remaining letters is selected at random.
4. The color of a car is selected at random; the type of transmission is selected at random.

#5-8 A and B are independent events. Find $P(A \text{ and } B)$. (1pt each)

5. $P(A) = 1/6$, $P(B) = 2/5$
6. $P(A) = 9/20$, $P(B) = 3/4$
7. $P(A) = .6$, $P(B) = .9$
8. $P(A) = 1/3$, $P(B) = 6/7$

#9-12, C and D are mutually exclusive events. Find $P(C \text{ or } D)$. (1pt each)

9. $P(C) = 2/5$, $P(D) = 3/5$
10. $P(C) = .5$, $P(D) = .375$
11. $P(C) = 12\%$, $P(D) = 27\%$
12. $P(C) = 5/8$, $P(D) = 1/8$

13. A and B are not mutually exclusive. If $P(A) = .5$, $P(B) = .25$, and $P(A \text{ and } B) = .125$, find $P(A \text{ or } B)$. (1pt)

#14-16, Two fair dice are rolled. State whether the events are mutually exclusive. (1pt each)

14. The sum is a prime number; the sum is less than 4.

15. The numbers are equal; the sum is odd.

16. The product is greater than 20; the product is a multiple of 3.

#17-22, A standard dice is tossed. Find each probability. (3pts each)

17. $P(3 \text{ or odd})$

18. $P(4 \text{ or even})$

19. $P(\text{even or less than } 4)$

20. $P(\text{odd or greater than } 2)$

21. $P(\text{odd or prime})$

22. $P(4 \text{ or less than } 6)$

#23-25, A jar contains four blue marbles and two red marbles. Suppose you choose a marble at random, and do not replace it. Then you chose a second marble. Find the probability of each event. (3pts each)

23. You select a blue marble and then a red marble.

24. You select a red marble and then a blue marble.

25. Both of the marbles you select are red.

KEY

Probability of Multiple Events

1. **Independent**
2. **Dependent**
3. **Dependent**
4. **Independent**
5. **$1/15$**
6. **$27/80$**
7. **.54**
8. **$2/7$**
9. **1**
10. **$7/8$**
11. **39%**
12. **$3/4$**
13. **$5/8$**
14. **Not mutually exclusive**
15. **Mutually exclusive**
16. **Not mutually exclusive**
17. **$1/2$**
18. **$1/2$**
19. **$5/6$**
20. **$5/6$**
21. **$2/3$**
22. **$5/6$**
23. **$4/15$**
24. **$4/15$**
25. **$1/15$**

Total points: 43

I. LESSON PLAN MODIFIED FOR BEHAVIOR DISORDERS

Lesson: Conditional Probability

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.2 - Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

Performance Objectives:

Students will determine the probability of simple events involving conditional probability with 80 percent accuracy when assigned homework (20 problems) from the textbook.

Assessment:

Students will be assigned homework from the textbook: **p. 700-701 #1-3, 6, 8-15, 18-20, 26-29, 33**. This homework will be due the next day at the beginning of class. The will be graded on work shown, how accurately they set up their probabilities, and correctness.

Advanced Preparation by Teacher:

3 small brown paper bags

Small candy bar

Candy bar wrapper

Procedure:

- *Intro* – Game Show Probability
 - The bags will be labeled 1, 2, and 3. In one bag, there will be a candy bar, one bag will have just a wrapper, and the other will be empty. The students should not know what is in the bags.
 - Have one student volunteer to be the “contestant”. Tell them that there is a special prize in one of the bags. They can have it if they choose correctly. Let them choose one of the bags.
 - Ask the class: What is the probability that the bag they have selected contains the prize. (1/3)
 - Before revealing what is in the selected bag, show them what is in one of the other bags (do not show them the bag containing the prize though).
 - After seeing what was in another bag, allow them the choice to pick again. Should they stick with their bag or select another??
 - This is an example of conditional probability.
- Have students get out their Math Journals and note taking materials.
- Finding Conditional Probability From a Table

- **Conditional Probability**

- Def.: The probability that an event B will occur, given that another event A, has already occurred.

- Example -

	Males (in thousands)	Females (in thousands)
Two-year colleges	1866	2462
Four-year colleges	4324	5517
Graduate schools	1349	1954

- The table shows students by gender at two- and four-year colleges, and graduate schools, in 2005. You pick a student at random.
 - P(female | graduate school)? ($\frac{1954}{3303} = .59$)
 - P(female)? ($\frac{2462+5517+1954}{1866+2462+4324+5517+1349+1954} = .57$)

- Using the Conditional Probability Formula

- Eqn.: $P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$
 - Example –

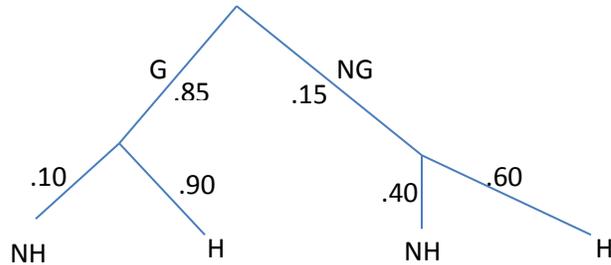
	Online	By Mail
Male	12	8
Female	24	6

- A utility company asked 50 of its customers whether they pay bills online or by mail. What is the probability that a customer pays the bill online, given that the customer is male?
 - P(male and online) = $\frac{12}{50}$
 - P(male) = $\frac{20}{50}$
 - P(online | male) = $\frac{\frac{12}{50}}{\frac{20}{50}} = .6$

- Using a Tree Diagram

- It follows from $P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$, that $P(A \text{ and } B) = P(A) \cdot P(B | A)$

- Tree diagrams help organize information.
- Example – In one high school, 85% of the students graduated in a particular class. Of the students who graduated, 90% are happy with their present jobs. Of the students who did not graduate, 60% are happy with their current jobs.



- What is the probability that a person from this class is happy with his or her present job?
 - $P(G \text{ and } H) = P(G) \cdot P(H|G) = .85(.90) = .765$

➤ Exit Slip

- Prompt – Describe how the intro activity relates to what you learned today.

Adaptations:

Behavioral Disorders

- When assigning the homework, I can be sure not to overload or overwhelm the student by breaking it up into shorter sections that still covers what they need to know.
- I can provide them with an activity check list before class to help them focus during the intro activity.
- I can give more specific instructions for the Exit Slip Prompt at the end of the lesson to ensure they understand what is being asked of them.

J. LESSON PLAN MODIFIED FOR AUTISM

Lesson: Analyzing Data

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.3 – Know and apply the characteristics of the normal distribution: identify setting in which the normal distribution may be useful; determine whether a set of data appears to be uniform, skewed, or normally distributed; use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two, or three standard deviations about the mean. (This standard does not closely tie with this lesson; however this lesson is necessary for understanding the next lesson in which this standard will better apply.)

Performance Objectives:

Students will analyze data with 85 percent accuracy when given a worksheet containing 4 different sets of data.

Assessment:

Students will be given a worksheet after the lesson. The worksheet will contain 4 different sets of data of which the students will analyze using methods learned in class. The students will be graded on work shown and correctness. This homework will be due at the beginning of class the next day.

Advanced Preparation by Teacher:

No preparation required.

Procedure:

- *Intro* – Arranging and Analyzing Data
 - Have students get into groups of 5. Once they are in their groups, ask them to arrange themselves by height (shortest to tallest) and record each person's height.
 - Have each group find the mean, median, and mode of their heights (this should be review).
 - Tell the students that they have just found measures of central tendency.
- Have students get out their Math Journals and note taking materials.
- Finding Measures of Central Tendency (Quick Review)
 - *Mean (Average)*
 - $$\frac{\text{sum of data values}}{\text{number of data values}}$$

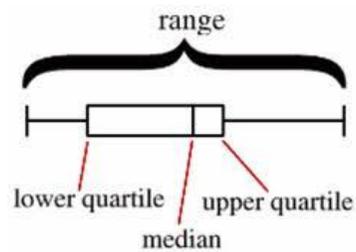
- *Median (Middle Number)*
 - When a data set is listed in order, the middle number. If there are two in the middle take the average of the two.
- *Mode (Most Often)*
 - The most frequently occurring value(s).
 - If there are two modes to a data set, the data set is called *bimodal*.
- Identifying an Outlier
 - An *outlier* is a value that is clearly different from the rest of the data in a set.
 - Ex. Find the outlier in the given data set: 56, 65, 73, 59, 98, 65, & 59.
 - After ordering the data we can clearly see that 98 is an outlier.
 - Outliers may cause statistical analysis of the data to be mi
- Comparing Data Sets
 - When looking at multiple data sets, you can compare the measures of central tendencies, range, quartiles, maximums, and minimums of each data set.
 - *Range* – difference between the greatest and least values.
 - *Quartiles* – the median divides the data set into two parts, and the median of each of these parts divides the data set into four parts called quartiles (these medians are labeled as Q_1 , Q_2 , Q_3).
 - *Interquartile range* – difference between the third and first quartiles.
 - *Maximum* – greatest value.
 - *Minimum* – least value.
 - Example – The table shows population density by square mile for counties in two of Florida’s eight regions, according to the 2000 U.S. Census.

Southwest	Southeast
204.2	228.1
13.7	573.0
548.6	1346.5
31.4	1157.9
124.1	79.8

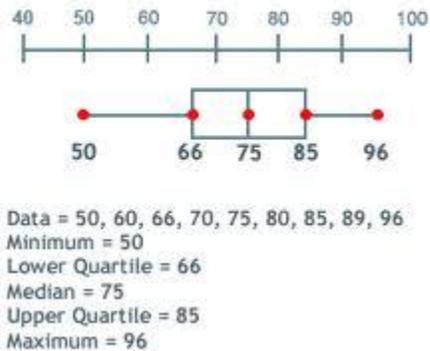
What are the mean, median, mode, and interquartile range for the Southwest and Southeast population density data? (Let the students work on this with their groups from the intro activity.)

- Southwest: mean = 184.4; mode = none; range = 534.9; $Q_1 = 22.55$; $Q_2 = 124.1$; $Q_3 = 376.4$; IQR = 353.85.
 - Southeast: mean = 677.06; mode = none; range = 1266.7; $Q_1 = 153.95$; $Q_2 = 573$; $Q_3 = 1252$; IQR = 1098.25.
- Using a **Box-and-Whisker Plot**
 - Def.: A way to display data that uses: quartiles to bound the center box and the minimum and maximum values to form the whiskers.

- Pic.:



- Ex.:



- *Closure* – Have the students make a box-and-whisker plot of the heights in their groups.
 - Pass out the worksheet.

Adaptations:

Autism

- When it is time to get into groups, I can make sure that this student is with others who have been taught to communicate effectively with them. It is also a good idea to educate the class about autism to create a better understanding.
- I can use illustrations to explain concepts and examples.
- Before the student comes to class, I can give them an overview or list of how things will go for this lesson.

Name: _____

Analyzing Data

For # 1 & 2, make a box-and-whisker-plot and find the mean for each set of values. Be sure to label each part.

1. 12 15 11 12 19 14 18 15 16

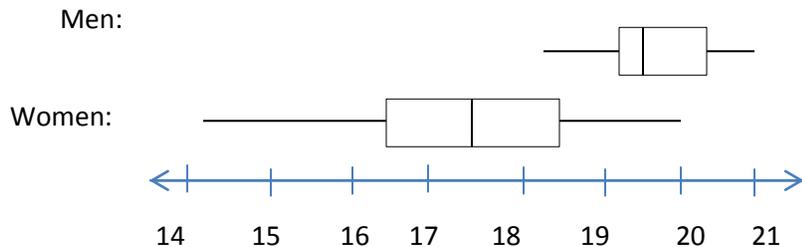
2. 120 145 133 105 117 150 130 136 128

For #3, identify the outlier. Then find the mean, median, and mode of the data set when the outlier is included and when it is not. What did you notice? Explain.

3. 87 104 381 215 174 199 233 186 142 228 9 53 117 129

For #4 you compare box-and-whisker plots.

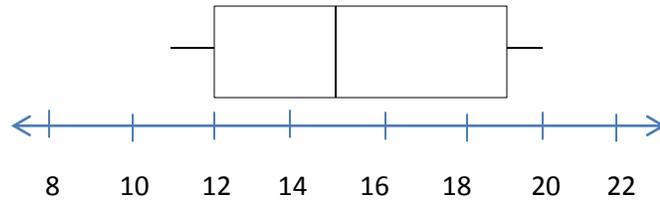
4. The box-and-whisker plots show the 36 best qualifying distances for the shot put events for men and women during the 2004 Olympics. Compare box-and-whisker plots. Describe any conclusions you can draw about Olympic-level male and female shot-putters.



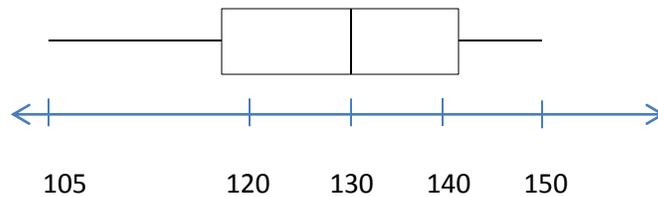
KEY

Analyzing Data

1. Mean: 14.7



2. Mean: 129.3



3. Outlier: 381; outlier included: mean – 161.2, median – 158, mode – none; outlier not included: mean – 144.3, median – 142, mode – none.

4. The range for the women's shot put is greater than that for the men's. The men are more consistent, as indicated by the shorter box and whiskers. Overall, the men tend to throw farther.

K. LESSON PLAN MODIFIED FOR MENTAL RETARDATION

Lesson: Standard Deviation

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.3 – Know and apply the characteristics of the normal distribution: identify setting in which the normal distribution may be useful; determine whether a set of data appears to be uniform, skewed, or normally distributed; use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two, or three standard deviations about the mean.

Performance Objectives:

Students will solve problems involving standard deviation and variation with 85 percent accuracy when assigned 18 problems from the textbook.

Assessment:

Students will be assigned homework from the textbook: **p. 715-717 #1-3, 6-13, 16-18, 20-23**. This homework will be due the next day at the beginning of class. The will be graded on work shown, how accurately they set up their probabilities, and correctness.

Advanced Preparation by Teacher:

Internet access and projector will be needed to show the clip for the intro.

Procedure:

- *Intro* – Statistics, Sports, and the Movies
 - Go to the website below to show the clip from the movie *Moneyball*.
 - <http://www.youtube.com/watch?v=FHG843nnabU&feature=related>
 - Shows a real-world application of statistics and probability.
 - Have the students make a list of 3-4 examples of how yesterday's lesson relates to the clip they just watched about statistics in baseball. Also have them write down a guess as to what standard deviation is.
- Have students get out their Math Journals and note taking materials.
- Definitions
 - **Standard Deviation**
 - Def.: measure of how far the numbers in a data set deviate from the mean.
 - Eqn.: $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$
 - **Variance**
 - Def.: the square of the standard deviation.

$$\blacksquare \text{ Eqn.: } \sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n}$$

➤ Finding Variance and Standard Deviation

- Find the mean, \bar{x} , of the n values in a data set.
- Find the difference, $x - \bar{x}$, between each value x and the mean.
- Square each difference, $(x - \bar{x})^2$.
- Find the average (mean) of these squares. This is the variance.
- Take the square root of the variance. This is the standard deviation.
- Example – What are the mean, variance, and standard deviation of these values: 6.9, 8.7, 7.6, 4.8, 9.0?
 - Mean: $\bar{x} = 7.4$
 - Variance: $\sigma^2 = 2.26$
 - Standard Deviation: $\sigma = 1.5$

➤ Using a Calculator to Find Standard Deviation

- Example – The table displays the number of U.S. hurricane strikes by decade from the years 1851-2000. What are the mean and standard deviation for this data?

Decade	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Strikes	19	15	20	22	21	18	21	13	19	24	17	14	12	15	14

Step 1 – Use **STAT EDIT** to enter the data in list **L1**.

Step 2 – In **STAT CALC** select the **1-Var Stats** option.

Answer: The mean is 17.6; the standard deviation is about 3.5.

➤ Using Standard Deviation to Describe Data

- Using the data from the previous example, within how many standard deviations from the mean do all of the values fall?
 - Draw a number line and plot the values.
 - Mark off the mean, and then mark off intervals of 3.5 on each side of the mean.
 - All of the values fall within 2 standard deviations of the mean.

- Have the students look back at their original guess for what standard deviation was. Ask them to compare their guess to the definition.
- Remind students that their Math Journals are due in two days. They will be able to use them to study for the test, but will turn them in before they take it.
- Assign the following for homework: **p. 715-717 #1-3, 6-13, 16-18, 20-23**. It is due tomorrow at the beginning of class.

Adaptations:

Mental Retardation

- First check with the special education teacher to make sure the objective of this lesson aligns the student's prioritized objectives. It may not be necessary that the student learns this lesson.

- Depending on how severe the student's mental retardation is, it may be necessary to modify the evaluation tool. Instead of completing the 18 problems for the textbook, I could create a worksheet that will effectively evaluate their prioritized objectives. Evaluation could even be done orally.
- I can use manipulatives to demonstrate the concept of standard deviation.

L. LESSON PLAN MODIFIED FOR SENSORY IMPAIRMENT

Lesson: Probability and Statistics Review

Length: 45 minutes

Age/Grade Intended: Algebra II

Academic Standard(s):

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

A2.8.2 – Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

A2.8.3 - Know and apply the characteristics of the normal distribution.

A2.8.4 - Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Performance Objectives:

- Students will completely fill out the given probability and statistics concept map.
- Working with a partner, students will participate in the review activity.

Assessment:

The students will be given a copy of the attached concept map and will fill it out completely. They will finish it in class and will receive 5 points for completing it.

Advanced Preparation by Teacher:

Make copies of the attached concept map and Connect 4 activity sheet and Connect 4 answer sheet. Hang up 4 copies of the Connect 4 answer sheet around the room.

Procedure:

- *Intro* – **Concept Map**
 - Allow students to work together to complete the concept map. This should take about 10 minutes.
- *Connect 4 Review Activity*
 - Pass out the Connect 4 activity sheet and explain the rules:
 - The goal is to get 4 spaces in a row.
 - Play rock-paper-scissors to see who will go first.
 - First player chooses a playable square (cannot play on a square unless the one below it is taken) and answers the question in the box. Then check the answer with one of the answer sheets hanging around the room.

- If you are right, color the square with your color. If you are wrong, the other player gets to try and steal it. If both players get the square wrong, the square is no ones, but if the square above it can still be played.
 - The first one to connect four square vertically or horizontally wins.
- Remind students that their Math Journals are due tomorrow. They also will take their unit test tomorrow. It will be 25 questions long. There will be 5 fill in the blank vocabulary questions, 3 multiple choice questions, and the rest will be computation. They should use their notes, Math Journal, homework, the concept map, book, and the Connect 4 activity worksheet to study for this test.

Adaptations:

Sensory Impairment

- If the student has poor eye sight, I make a copy of the concept map and Connect 4 activity sheet that has larger and bold font.
- Also if the student has visual impairment, I can make sure to partner them with another student who would be willing to read/explain the concept map and Connect 4 activity sheet.
- For hearing impaired students, I can make sure to partner this student with someone they can communicate well with.
- I can also provide the hearing impaired student with written instructions for both the concept map and Connect 4 activity.

* **NOTE:** The version of the Probability and Statistics concept map I would use would be in landscape orientation. I was unable to insert a landscape document into this unit plan with portrait documents without changing everything to landscape orientation. I modified the concept map so that it would fit in this unit plan document.

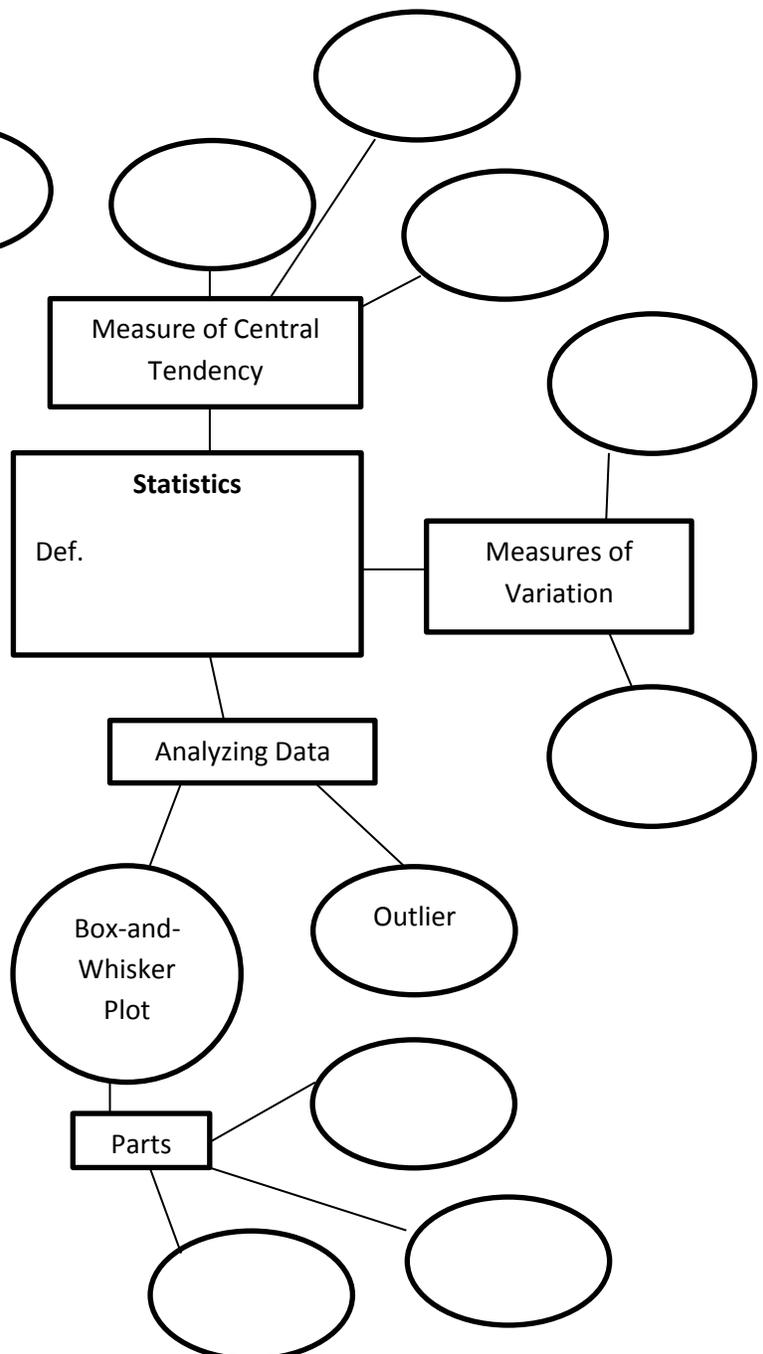
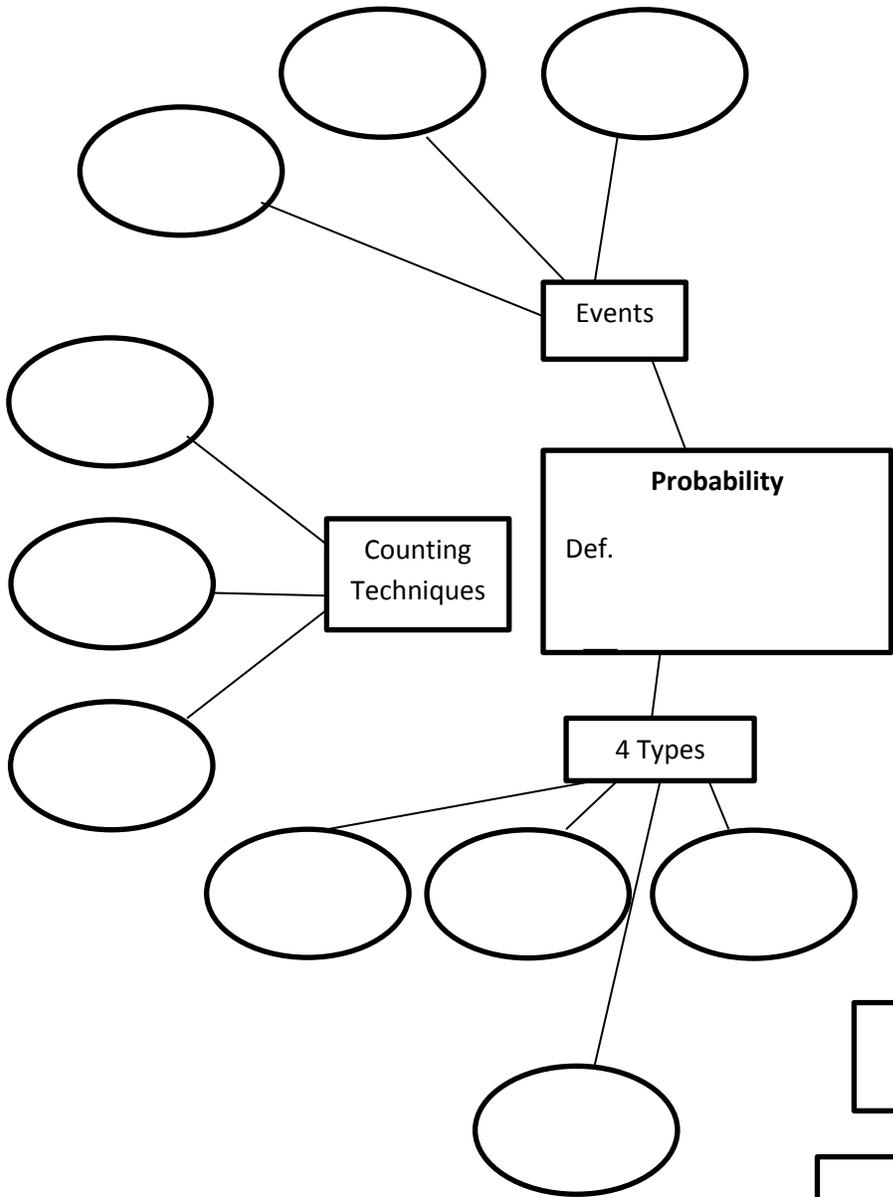
Connect 4!

<p>What is the range of: 1 16 23 4 5 11 14 5 2 2 8 9 2 16</p>	<p>What kind of animal is on the cover of our textbook?</p>	<p>What is the formula for standard deviation?</p>	<p>A coin is tossed and recorded 161 heads and 179 tails. What is the experimental probability of head?</p>	<p>The outcome of one event does not affect the outcome of another event.</p>	<p>What is the variance of: 6.5 5.8 3.9 5.7 4.2</p>
<p>What is the variance of: 6.5 5.8 3.9 5.7 4.2</p>	<p>A value that is substantially different from the rest of the data.</p>	<p>The number that occurs most often in a data set.</p>	<p>What is Miss Fauser's favorite color?</p>	<p>Three students are selected from a class of 12 to be president, vice-president, and secretary. How many ways can these spots be filled?</p>	<p>Used when order is not important.</p>
<p>What is the second planet from the sun?</p>	<p>${}_8C_5$</p>	<p>If $P(A) = .6$ and $P(B) = .25$, what is $P(A \text{ and } B)$?</p>	<p>Within how many standard deviations of the mean do all of the data values fall: 12 17 15 13 9 10 15 17</p>	<p>What does it mean for a data set to be bimodal?</p>	<p>A way to display data using quartiles.</p>
<p>The difference between the greatest and least values.</p>	<p>Find the mode of: 1 4 6 3 7 3 8 8 9 2 8 0 1 2 4 8 9 3 0</p>	<p>Type of theoretical probability involved in geometry problems.</p>	<p>8!</p>	<p>What kind of dog is Scooby Doo?</p>	<p>What does standard deviation measure?</p>
<p>An arrangement of items in a particular order.</p>	<p>What is sample space?</p>	<p>What are the measures of central tendency?</p>	<p>What effect does an outlier have on the standard deviation of a data set?</p>	<p>${}_7C_5$</p>	<p>If $P(A) = .6$ and $P(B) = .25$, what is $P(A B)$?</p>
<p>How many ways are there to select 5 actors from a troupe of nine to improvise a scene?</p>	<p>${}_6P_3$</p>	<p>What is the equation for conditional probability?</p>	<p>$\frac{15!}{10! 5!}$</p>	<p>P(rolling an even number on a six sided dice)</p>	<p>Find the mean of: 1 2 3 3 4 5 5 9</p>

Connect 4! Answers

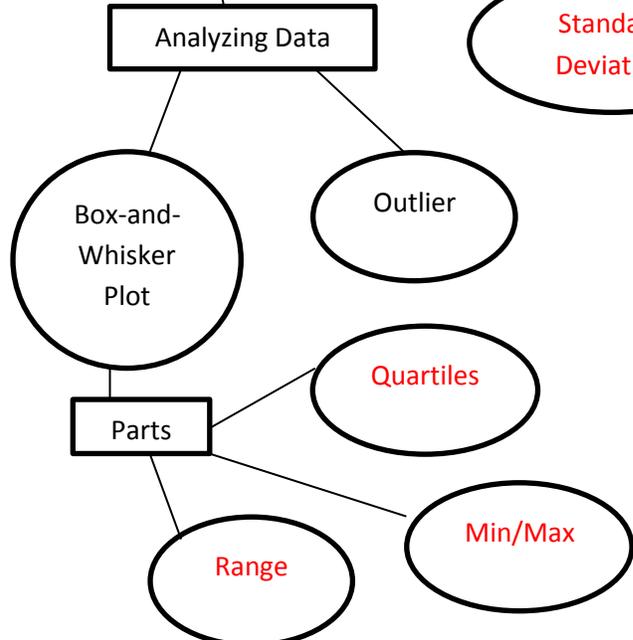
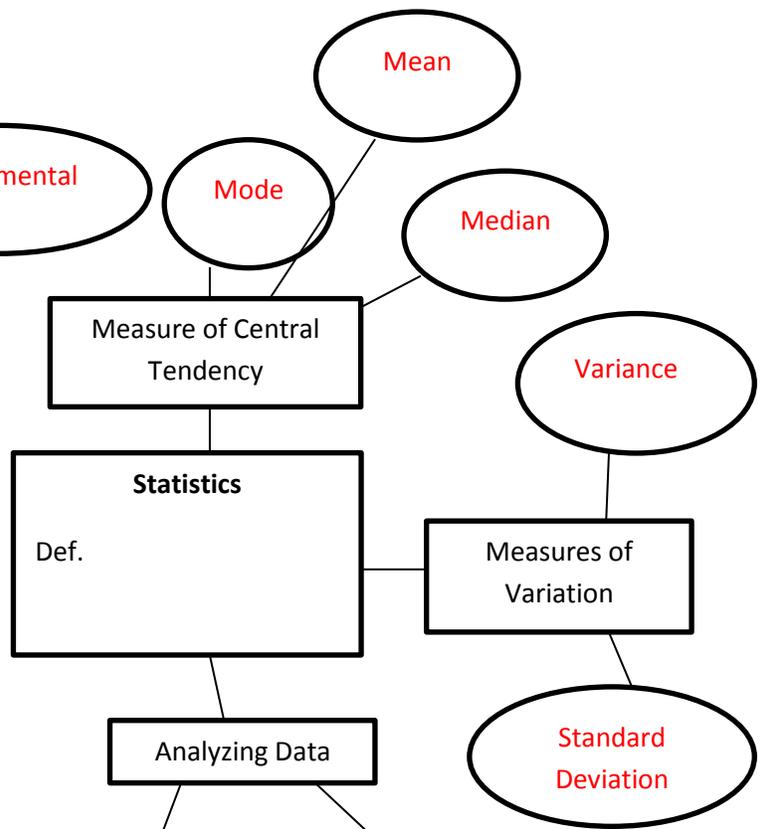
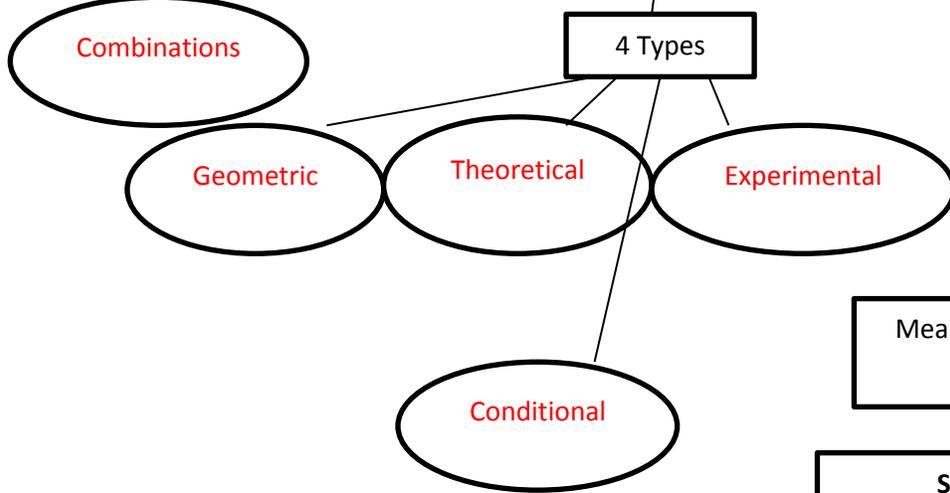
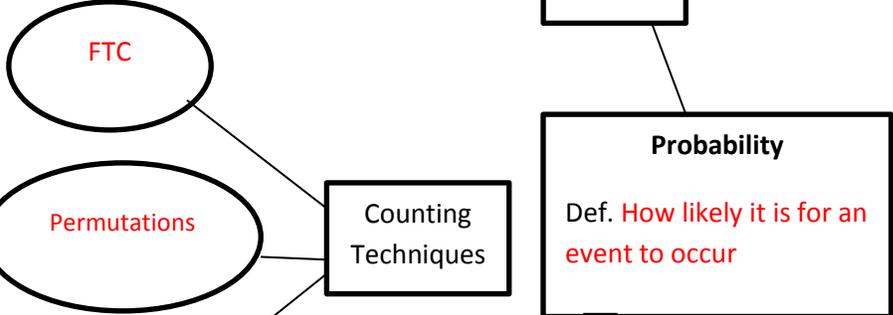
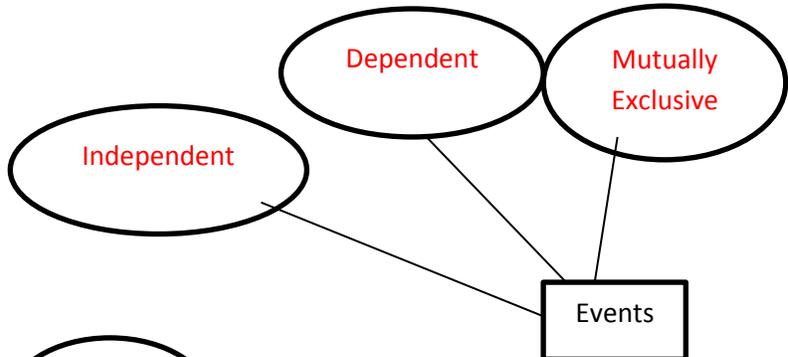
22	Lizard	$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$.47	Independent Events	.9976
.999	Outlier	Mode	Purple	(Permutation) 1320	Combination
Venus	56	.15	2 standard deviations	The data set has 2 modes.	Box-and-whisker plot
Range	8	Geometric Probability	40320	Great Dane	How far the data values in a set deviate from the mean.
Permutation	The set of all possible outcomes	Mean, Median, & Mode	Increases the standard deviation	21	.6
(Combination) 36 ways	120	$P(B A) = \frac{P(A \text{ and } B)}{P(A)}$	3003	.5	4

Probability



Statistics

Probability



Statistics

M. UNIT TEST & MODIFIED TEST WITH ANSWER KEYS

UNIT TEST – PROBABILITY & STATISTICS

Standards Covered:

A2.8.1 - Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

A2.8.2 – Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

A2.8.3 - Know and apply the characteristics of the normal distribution.

A2.8.4 - Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Instructions:

- Before passing out the test, collect the students' Math Journals.
- Ask the students if they have any remaining questions, and answer any they may have.
- Pass out the test.
- The students will work individually on the test and will not be allowed to use any notes. They can use their calculators.
- They will have the entire class period to take the test.

Modifications:

Learning Disabilities

- Remove redundant questions.
- Limit the number of options for the multiple choice questions.
- Provide a word bank for the fill-in vocabulary questions.
- Modify the final question to make it appropriate.

When you have students with learning disabilities in your classroom it may be necessary to alter assessment material and methods. It is important to communicate with the special education teacher to be sure you make the appropriate modifications for the student according to their IEP. Modifying assessment does not mean making it easier. Instead, it means arranging the material in a way that allows the student to be successful, but yet still assuring they are meeting the standards they need to meet.

The modified version of this test is specifically designed for students with Learning Disabilities. I limited the number of questions, but made sure that it still adequately assessed mastery of the standards. Making sure not to overwhelm a student with LD, I also limited the number of options on the multiple choice section. To ensure the opportunity for success of this student, I provided a word bank for the fill-in vocabulary portion of the test. The final question of the test is a short answer question. On the modified version, I chose a more appropriate question on one of the main topics of this unit. This test could also be read aloud to the student if necessary.

Name: _____

Score: ____/40

Probability & Statistic Unit Test

Directions: You may use a calculator to complete the test, but be sure to show all your work when necessary. Partial credit is possible, and points are list in parentheses.

Vocabulary – Fill-in the blank with the appropriate vocabulary word. (1 point each)

1. If each outcome has the same chance of occurring, then the events are _____.
2. The probability that an event, B, will occur given that another event, A, has already occurred is called a _____.
3. A _____ is an arrangement of items in a particular order.
4. _____ measures how likely it is for an event to occur.
5. Two events are _____ if the occurrence of one event affects the probability of the second event.

Multiple Choice – Circle the best answer. (1 point each)

6. A value that is substantially different from the other data values is called the _____?
 - a. Mode
 - b. Outlier
 - c. Median
 - d. Interquartile range
7. A selection in which order does not matter is called _____.
 - a. Combination
 - b. Fundamental Counting Principle
 - c. Mutually Exclusive
 - d. Permutation
8. The mean of a data set is also known as the _____.
 - a. Median
 - b. Range
 - c. Average
 - d. Standard Deviation

Computation – Read the questions carefully and be sure to show all work.

Evaluate each of the following (2 points each):

9. $\frac{5!}{2!2!}$

10. ${}_4C_3 + {}_6C_5$

11. ${}_4P_3 + {}_6P_5$

Q and R are independent events. Find P(A and B). (2 points each)

12. $P(Q) = 0.5, P(R) = 0.4$

13. $P(Q) = \frac{1}{3}, P(Q) = \frac{3}{8}$

Use the table for #14-16. (2 points each)

Age of Respondent	Number of Groups	
	0-4	5 or more
< 30	7	18
≥ 30	12	12

14. Find $P(5 \text{ or more})$.

15. Find $P(5 \text{ or more} \mid \text{age} < 30)$.

16. Find $P(\text{age} \geq 30 \mid 0-4)$

Two standard dice are tossed. For #17 and #18, state whether the events are mutually exclusive. (1 point each)

17. One of the numbers is 1 less than the other. The sum is odd.

18. The sum is greater than 10. Six is one of the numbers.

Find the mean, median, mode, Q_1 , and Q_3 for #19. (5 points)

19. 36 38 42 47 51 56 62 69 70 74

20. On a camping trip, you bring 12 items for 4 dinners. For each dinner, you use 3 items. In how many ways can you choose the 3 items for dinner for the first dinner? For the second dinner? (4 points)

Short Answer – Answer the following question using complete and proper sentences. (5 points)

21. How are measures of central tendency different from standard deviation?

KEY

Probability & Statistic Unit Test

1. Equally likely
2. Conditional Probability
3. Permutation
4. Probability
5. Dependent
6. B
7. A
8. C
9. 30
10. 10
11. 744
12. 0.2
13. $1/8$
14. $30/49$
15. $18/25$
16. $12/19$
17. Not mutually exclusive
18. Not mutually exclusive
19. Mean: 54.7, Median: 53.5, Mode: no mode, Q_1 : 42, Q_2 : 69
20. 220, 84
21. Sample answer: Measures of central tendency are ways to describe a “middle” value of a data set. Standard deviation describes how data are spread out from a particular middle value.

Name: _____

Score: ____/35

Probability & Statistic Unit Test (Modified)

Directions: You may use a calculator to complete the test, but be sure to show all your work when necessary. Partial credit is possible, and points are list in parentheses.

Vocabulary – Fill-in the blank with the appropriate vocabulary word. (1 point each)

Word Bank:

Conditional Probability Equally Likely Probability Dependent Permutation

1. If each outcome has the same chance of occurring, then the events are _____.
2. The probability that an event, B, will occur given that another event, A, has already occurred is called a _____.
3. A _____ is an arrangement of items in a particular order.
4. _____ measures how likely it is for an event to occur.
5. Two events are _____ if the occurrence of one event affects the probability of the second event.

Multiple Choice – Circle the best answer. (1 point each)

6. A value that is substantially different from the other data values is called the _____?
 - a. Outlier
 - b. Median
 - c. Interquartile range
7. A selection in which order does not matter is called _____.
 - a. Combination
 - b. Fundamental Counting Principle
 - c. Permutation
8. The mean of a data set is also known as the _____.
 - a. Median
 - b. Range
 - c. Average

Computation – Read the questions carefully and be sure to show all work.

Evaluate each of the following (2 points each):

9. $3!$

10. 7C_2

11. 6P_2

Q and R are independent events. Find $P(A \text{ and } B)$. (2 points)

12. $P(Q) = 0.5$, $P(R) = 0.4$

Use the table for #13 & #14. (2 points each)

Age of Respondent	Number of Groups	
	0-4	5 or more
< 30	7	18
≥ 30	12	12

13. Find $P(5 \text{ or more})$.

14. Find $P(\text{age} \geq 30 \mid 0-4)$

Two standard dice are tossed. For #15, state whether the events are mutually exclusive. (1 point)

15. One of the numbers is 1 less than the other. The sum is odd.

Find the mean, median, mode, Q_1 , and Q_3 for #19. (5 points)

16. 36 38 42 47 51 56 62 69 70 74

a. Mean –

b. Median –

c. Mode –

d. Q_1 –

e. Q_2 –

17. On a camping trip, you bring 12 items for 4 dinners. For each dinner, you use 3 items. In how many ways can you choose the 3 items for dinner for the first dinner? For the second dinner? (4 points)

Short Answer – Answer the following question using complete and proper sentences. (5 points)

18. List and describe the three measures of central tendency.

KEY

Probability & Statistic Unit Test

- 1. Equally likely**
- 2. Conditional Probability**
- 3. Permutation**
- 4. Probability**
- 5. Dependent**
- 6. A**
- 7. A**
- 8. C**
- 9. 6**
- 10. 21**
- 11. 30**
- 12. 0.2**
- 13. 30/49**
- 14. 12/19**
- 15. Not mutually exclusive**
- 16. Mean: 54.7, Median: 53.5, Mode: no mode, Q_1 : 42, Q_2 : 69**
- 17. 220, 84**
- 18. Sample answer: The mean is the average of a data set. The mode is the value that occurs most often in a data set. The median is the number in the middle when a data set is arranged in order from least to greatest.**

N. REFLECTION PAPER

Jessica Fauser
EDUC 352
December 8, 2011
Unit Plan Reflection

Unit Reflection

Reading in a mathematics class is nearly non-existent. Even I, who has always had an interest in math, never had much practice in reading a math textbook until I got to college. From the experience I have gained through this Adolescent Exceptional Learners course, I have come to realize how important reading for math is to the learning process.

For those who math comes easy to, reading the textbook is often not necessary for comprehending the material. In a traditional mathematics classroom, everything is done by example. The teacher shows you how to compute the problems; you copy the examples in your notebook, and then complete the assignment. Students get by without ever opening their textbooks, except to do the assigned homework. This was the case for just about every math class I took in high school. I have also observed this style of teaching in a number of field placements I have been assigned. It appears to be effective in some cases, but what about the student for which math does not come easy? What about students with exceptionalities? Unfortunately, these students are left behind in this style of teaching.

Students are not being taught how to use their books and notes as resources. I have tutored many students who have asked me questions for which I showed them the answer word-for-word right out of their book or class notes. I will admit there were some concepts in high school that I did not fully understand until later. I only knew how to do things just because it was what my teacher showed me to do. For example, I had no clue what the significance of a derivative was, but I knew how to compute it. After taking a calculus course in college, that had

assigned readings, I finally knew what a derivative was. Looking back now, had I read my book in high school, I would have known what it meant then. However, reading in math, especially at the high school level, is not emphasized.

Math textbooks are different than those of other subjects. Typically, information is broken up into short paragraphs and followed by an example or two to illustrate. It is often difficult for students to sort through the given information to find what is important and what is not. As a math teacher, it is our responsibility to teach students how to read a math textbook. We have to help them develop their reading skills so that they can be successful, not only in mathematics but in other subjects as well. It will also help prepare them for higher education courses. The first couple math courses I took at Manchester College were challenging for me because I did not know how to use my textbook. It was expected that we read our books, but I did not think it was that important. I learned the hard way that this was not true. I hope to be able to show my students the importance of reading their textbooks.

One reading strategy that will be most beneficial in math is rereading. Reading a section before the teacher gives a lesson lets the student get an idea of what they will be expected to know. It also allows them to see what they already know and gives them a chance to prepare questions for what they do not understand. When the student rereads the material after the lesson is given, they will be able to sort through the information and determine what is important. Encouraging students to reread the notes they take will also be helpful. Looking over their notes will help the students make important connections and give them the opportunity to see where they are struggling. If you show the students how to use their books and notes, they will not be so dependent on the teacher. I noticed this in my high school placement. During homework

time, I observed that when the students got stuck, they would immediately ask the teacher for help, rather than first checking their book or notes.

There are a variety of ways I can implement reading and the rereading strategy in my classroom. It is not enough to just assign the reading. You have to hold the students accountable. One way I can hold my future students accountable is to have short reading quizzes at the beginning of class. Another strategy to get the students to read and think about the material is to have them write down a few questions about what they did not understand. Then, after the lesson and rereading the material, have them see if their questions were answered. In one of the math courses I took at Manchester College, our professor had a unique way of holding us responsible for reading the material. Everyone in class was all assigned a number. At the beginning of each class, our professor would roll the die. If our number was rolled, we had to present the material we were assigned to read for the lesson that day. I could certainly do something similar to this in my future classroom to get the students to read.

Teaching students how to read their math textbooks and their notes will have an impact on the learning process. I hope to change my future students' views on reading in math, and to show them what they can gain from knowing how to read their textbooks. I have learned through personal experience and from experience in this class that reading is just as important in math as it is in other subjects such as English, science, and social studies. Emphasizing the importance of reading early will help set the students up for success in the math classroom and will help prepare them for future courses they may take.