Deductive Logic

Overview

(1) Distinguishing Deductive and Inductive Logic

(2) Validity and Soundness

(3) A Few Practice Deductive Arguments

(4) Testing for Invalidity

(5) Practice Exercises

Deductive and Inductive Logic

Deductive vs Inductive

Deductive Reasoning

- Formal (the inference can be assessed from the form alone).
- When sound, the conclusion is *guaranteed* to be true.
- The conclusion **is extracted from** the premises.

Inductive Reasoning

- Informal (the inference *cannot* be assessed by the form alone).
- When cogent, the conclusion is only probably true.
- The conclusion **projects beyond** the premises.

Review of Basic Terms

Deductive Logic: Basic Terms

Validity

- A property of the **form** of the argument.
- If an argument is valid, then the truth of the premises guarantees the truth of the conclusion.

Soundness

- A property of the **entire argument**.
- If an argument is sound, then:
  - (1) it is valid, and
  - (2) all of its premises are true.

Validity

If an argument is **valid**, then the truth of the premises guarantees the truth of the conclusion.

A valid argument can have:

- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

A valid argument can not have:

- True premises, false conclusion

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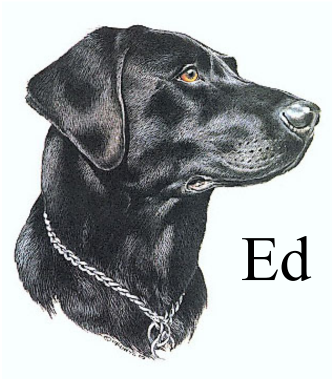
All dogs are mammals.
Ed is a dog.
∴ Ed is a mammal.



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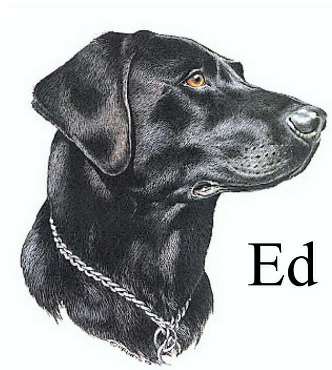
A valid argument can have:
- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

A valid argument can not have:
- True premises, false conclusion

All cats are dogs.
Ed is a cat.
∴ Ed is a dog.



Ed

Validity

If an argument is **valid**, then the truth of the premises guarantees the truth of the conclusion.

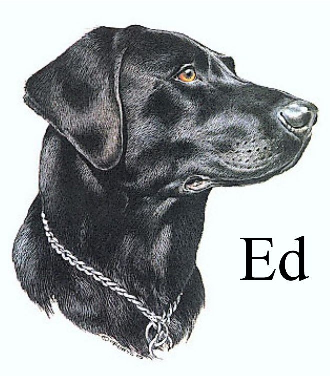
A valid argument can have:

- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

A valid argument can not have:

- True premises, false conclusion

All cats are toads.
Ed is a cat.
∴ Ed is a toad.



Sample Deductive Arguments

Deductive Argument #1

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s not raining.

∴ (3) You won’t need your umbrella.

Checking for Invalidity

Two Methods of Counter-example

Alternate scenario

Imagine some **alternate scenario** in which the premises of the argument will be true, but the conclusion false.

Substitution (two-step)

(1) Determine the form of the argument.

(2) Substitute other statements, such that all the premises will be true but the conclusion false.

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(2) It's not raining.

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(1)	If R, then U	R = I'm a dog.
(2)	Not-R	U = I'm a mammal.

∴ (3) Not-U

[Denying the Antecedent]

INVALID

Deductive Argument #2

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s raining.

∴ (3) You’ll need your umbrella.

Deductive Argument #2

(1) If it's raining, then you'll need your umbrella.

(2) It's raining.

∴ (3) You'll need your umbrella.

(1)	If R, then U	If P, then Q
(2)	R	P
∴ (3) U		∴ Q

[Modus Ponens (Latin: "mode that affirms")]

VALID

Deductive Argument #3

If Ed has black hair, then Ed is Italian.

Ed does have black hair, so Ed is Italian.

Deductive Argument #3

If Ed has black hair, then Ed is Italian.

Ed does have black hair, so Ed is Italian.

(1) If B, then I

(2) B

∴ (3) I

[Modus Ponens]

VALID

Ed

Deductive Argument #4

If God exists, then there's no evil in the world. But there *is* evil in the world, so God must not exist.

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If God exists, then there's no evil in the world. But there *is* evil in the world, so God must not exist.

(1) If G, then not-E	If P, then Q
(2) E	not-Q
∴ (3) not-G	∴ not-P

[Modus Tollens (Latin: "mode that denies")

VALID

Deductive Argument #5

If the medicine doesn't work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.

Deductive Argument #5

If the medicine doesn't work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.

(1)	If not-W, then D	If P, then Q
(2)	D	Q
∴ (3)	not-W	∴ P

[Affirming the Consequent]

INVALID

Deductive Argument #6

That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.

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That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.

(1) J or M	P or Q
(2) not-J	not-P
∴ (3) M	∴ Q

[Disjunctive Syllogism]

VALID

Practice Argument #1

If he was lost, then he would have asked for directions. But he didn't ask for directions. So he must not be lost.

(1) If L, then D	If P, then Q
(2) not-D	not-Q
∴ (3) not-L	∴ not-P

[Modus tollens]

VALID

Practice Argument #2

If interest rates drop, then the dollar will weaken against the Euro. Interest rates did drop. Therefore, the dollar will weaken against the Euro.

(1) If I, then D	If P, then Q
(2) I	P
∴ (3) D	∴ Q

[Modus ponens]

VALID

Practice Argument #3

If his light is on, then he's home. But his light isn't on, so he's not home.

(1)	If L, then H	If P, then Q
(2)	not-L	not-P
∴ (3)	not-H	∴ not-Q

[Denying the Antecedent]

INVALID

Practice Argument #4

The mind is an immaterial substance, for it is either identical to the brain or it is an immaterial substance, and it's not identical to the brain.

(1) B or I	P or Q
(2) not-B	not-Q
∴ (3) I	∴ P

[Disjunctive Syllogism]

VALID

Practice Argument #5

If you want to get into law school, then you'd better do your logic homework.

(1)	If L, then H	If P, then Q
[(2)	L]	P
[∴(3)	H]	∴ Q

[Enthymeme, expanded as modus ponens]

VALID

Determining Validity

To determine invalidity…

… we can use the method of counter-example.

To determine validity…

… we need something else: Truth Tables

Truth Tables

Example

(1) If I win the lottery, then I'll buy you dinner.. If p, then q

(2) I won the lottery.. p

(3) I'll buy you dinner. ∴ q

	p	q	if p, then q	p	q
1	T	T	T	T	T
2	T	F	F	T	F
3	F	T	T	F	T
4	F	F	T	F	F
			P1	P2	C

indicates validity

Why do conditionals have these truth-values?

Truth Tables

Example

(1) If it's raining, then you'll need your umbrella. If p, then q

(2) It's not raining. not-p

(3) You don't need your umbrella. ∴ not- q

	p	q	if p, then q	–p	–q
1	T	T	T	F	F
2	T	F	F	F	T
3	F	T	T	T	F
4	F	F	T	T	T
			P1	P2	C

indicates invalidity

The TV of Conditionals

The logic of conditional statements is such that they are false only when the antecedent is true and the consequent is false.

A= **If I win the lottery, then I'll buy you dinner.**

Suppose…

(1) I both win the lottery and buy you dinner. (A is true)

(2) I win the lottery, but don't buy you dinner. (A is false)

(3) I lose the lottery, but still buy you dinner. (A is true)

(4) I lose the lottery, and don't buy you dinner. (A is true)

“Or”

In **English**, ‘or’ can be used either inclusively or exclusively:

Inclusive “or”: “P or Q or both”

Example: “He’s either reading a book or out in the garden (or both).”

Exclusive “or”: “P or Q but not both”

Example: “The train’s coming in on either platform 3 or platform 5.”

In **logic**, “or” is always understood in the inclusive sense.

Inductive Logic

Inductive Logic Overview

(1) Distinguish inductive from deductive arguments

(2) Define ‘strength’ and ‘cogency’

(3) Describe four kinds of inductive arguments

(4) Practice exercises on inductive logic

Deductive and Inductive Arguments

Deductive vs Inductive

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Strength and Cogency

Strength

- A property of the argument.
- If an argument is strong, then the truth of the premises guarantees the *probable* truth of the conclusion.
- Unlike validity (which is all or nothing), inductive strength comes in degrees, and is determined by the content of the premises.

Cogency

- A property of the argument.
- If an argument is cogent, then (1) it is strong, and (2) all of its premises are true.

Strength and Cogency

Deductive Logic

- **Valid/Invalid inferences**
- **Sound/Unsound arguments**
- **Conclusions are *guaranteed* true.**

Inductive Logic

- **Strong/Weak inferences**
- **Cogent/Uncogent arguments**
- **Conclusions are *probably* true.**

Sample Inductive Arguments

Common Inductive Reasoning

Generalization

Inferring your beliefs about the whole of X from a part of X.

Authority

Inferring your beliefs about X from the beliefs held by a trusted source.

Analogy

Inferring your beliefs about a lesser known thing from its similarities with a better known thing.

Hypothetical Induction

Discovering the best explanation for some thing or event.

Agument from Generalization

(1) Token 1 of type A has property X.

(2) Token 2 of type A has property X.

∴ (3) All tokens of type A have property X. [or]

∴ (3’) The next token of type A will have property X.

Token = an individual
Type = a class of individuals

Agument from Generalization

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Token = an individual
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∴ (3) All tokens of type A have property X. [or]

∴ (3') The next token of type A will have property X.

Example: (1) The first Twinkie I ate from this box of 24 Twinkies had its créme filling riddled with mouse droppings. (2) The second Twinkie I ate from the box was similarly contaminated. (3) Likewise with the third Twinkie. (4) Therefore, the next (fourth) Twinkie I eat from this box will likely also have mouse droppings in its crème filling.

Argument from Authority

(1) S (some person) is a reliable authority regarding P (some statement).

(2) S believes P.

∴ (3) P.

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Example: (1) Ed Smith has a Ph.D in physics, and (2) he believes that objects fall at the same rate regardless of weight, once air resistance is taken into account. (3) Therefore, it's probably true that objects fall like that.

Argument from Analogy

(1) Items A and B have property X.

(2) A also has property Y.

∴ (3) B also has Y.

A = Primary Analogate
B = Secondary Analogate
X = shared property that is known.
Y = shared property that is inferred.

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X = shared property that is known.
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Example: (1) Jane and Nancy both received National Merit Scholarships, both are math majors, and both like Dr. Rich's classes. (2) Jane received an A in Dr. Rich's Calculus II class last semester, so (3) Nancy should do equally well in the same class this coming semester.

Six Rules for Strong Analogies

(1) **Relevance**: Relevance of the known shared property (x) to the inferred shared property (y). [The more relevant, the stronger the analogy.]

(2) **Disanalogy**: Nature and degree of disanalogy, i.e., differences between the primary and secondary analogates. [Usually, the more disanalogies, the weaker the analogy.]

(3) **Similarities**: Number of similarities between primary and secondary analogates. [The more similarities, the stronger the analogy.]

(4) **Sample size**: Number and kind of primary analogates. [The more samples, the stronger the analogy.]

(5) **Sample diversity**: Diversity among the primary analogates; randomized sampling strengthens the likelihood of the secondary analogate sharing the contested property. [Usually, the greater the number, the stronger the analogy.]

(6) **Specificity**: Specificity of the conclusion relative to the premises. [The more specific the conclusion, the weaker the analogy.]

Analogy: Applying the Rules

Jane and Nancy both received National Merit Scholarships, both are math majors, and both like Dr. Rich's classes. Jane received an A in Dr. Rich's Calculus II class last semester, so Nancy should do equally well in the same class this coming semester.

(1) Relevance All three stated similarities are relevant. [strengthens]

(2) Disanalogy Suppose Jane's SAT math was 800, while Nancy's was only 450. [weakens]

(3) Similarities Suppose there are more similarities, e.g., Jane and Nancy have always performed equally well in their math classes. [strengthens]

(4) Sample size Suppose Al, Betty, and Carl also share these same properties with Jane. [strengthens]

(5) Sample diversity Suppose all four differ on many other characteristics, some of which Nancy shares, others she does not. [strengthens]

(6) Specificity We change the claim to: Nancy will get at least a B. [strengthens]

Hypothetical Induction

(abduction)

(1) P (some surprising phenomenon).

(2) If H (some hypothesis), then P.

(3) H is the best available explanation of P.

$\therefore$ (4) H.

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Example: (1) The left-over pizza has been eaten. (2) If John stopped by, then he would have eaten it. (3) I can’t think of anyone else who might have eaten my pizza without asking. Therefore, (4) John must have stopped by and eaten my pizza.

Explanation to the Best Inference

Hypothetical induction (abduction; explanation to the best inference) is the positing of some **theoretical entity** or **structure** in order to explain some observed phenomenon (a "**surprising fact**").

The hypothesis is meant to **explain** the observed phenomenon, so that if the explanation were *true* then the fact would no longer be surprising.

Practice Arguments

Practice Argument #1

Every time I eat at Ed's Diner, the coffee has been wretched, so the coffee will likely be wretched today as well.

Practice Argument #1

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Generalization, strong.

Practice Argument #2

This lovely china plate is similar in size, weight, and composition to the one I just dropped on your head, and that one broke. Therefore, it stands to reason that when I drop this plate on your head, it too will break.

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Analogy, strong.

Practice Argument #3

"A dog was kept in the stables, and yet, though someone had been in and had fetched out a horse, he had not barked enough to arouse the two lads in the loft. Obviously the midnight visitor was someone whom the dog knew well." [Arthur Conan Doyle, *Memoirs of Sherlock Holmes*]

Practice Argument #3

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Hypothetical

Practice Argument #4

Jerry Lewis just said on television that global warming is a serious environmental issue, so I guess it must be.

Practice Argument #4

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Authority, weak.

Practice Argument #5

A porpoise is similar to a human being. It has lungs rather than gills. It is warm-blooded rather than cold-blooded. And porpoises nurse their young with milk. Therefore, porpoises, like humans, are probably capable of speaking languages.

Practice Argument #5

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Analogy, weak.

Practice Argument #6

The *Journal-Gazette* reported recently that three teenagers were arrested on drug possession. Teenagers these days are nothing but a bunch of junkies.

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Generalization, weak.

(Hasty Generalization)

Practice Argument #7

Dr. Blithers, an internationally respected paleontologist, told me that the massive dinosaur die-off was most likely the result of an asteroid colliding with the earth. What's more, this hypothesis enjoys widespread support in the scientific community. So my guess is that it's true.

Practice Argument #7

Dr. Blithers, an internationally respected paleontologist, told me that the massive dinosaur die-off was most likely the result of an asteroid colliding with the earth. What's more, this hypothesis enjoys widespread support in the scientific community. So my guess is that it's true.

Authority, strong.

Practice Argument #8

That porcupine climbing up your leg is similar in size and age to the one you found climbing up your leg yesterday. Likewise, it's behaving in the same odd manner: swaying head, frothing mouth, and a peculiar whistling sound coming from its nose. I bet this porcupine, if left to its own devices, will bite you in the neck just like the one yesterday.

Practice Argument #8

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Analogy, strong.

Practice Argument #9

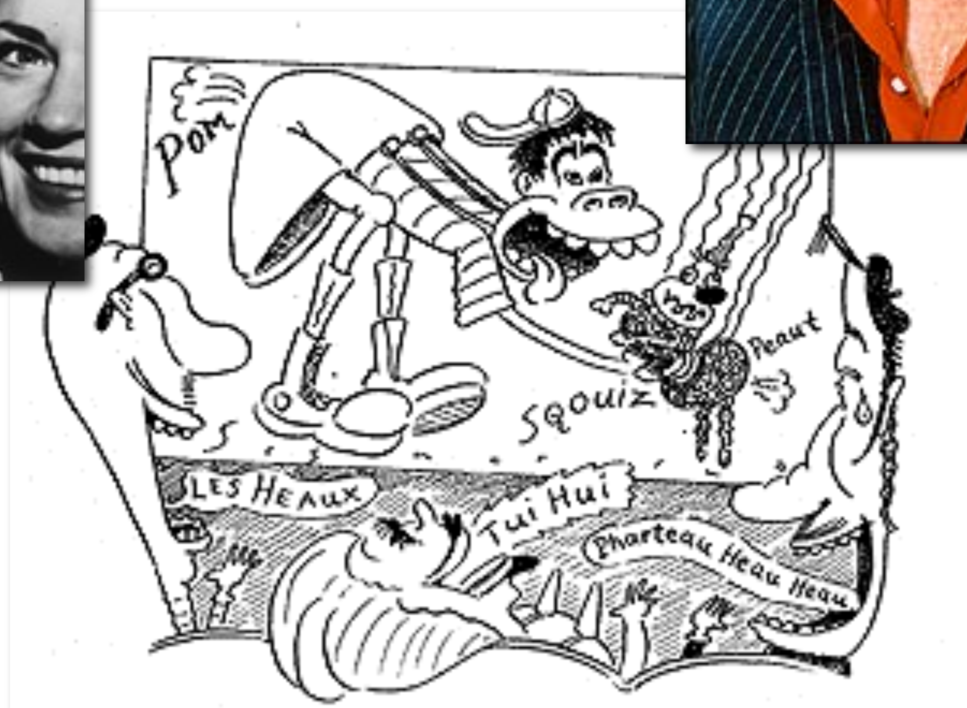
Every time I hear the garbage can tip over and I run out to check on it, I discover a raccoon inside the can looking for some dinner. That was the can falling over just now, so I suspect we'll find a raccoon if we go out and check.

Practice Argument #9

Every time I hear the garbage can tip over and I run out to check on it, I discover a raccoon inside the can looking for some dinner. That was the can falling over just now, so I suspect we'll find a raccoon if we go out and check.

Generalization, strong

Jerry Lewis?



Do the French really love Jerry Lewis? Yes.

http://www.straightdope.com/classics/a991001.html