# Deductive Logic

# Overview

(1) Distinguishing Deductive and Inductive Logic

(2) Validity and Soundness

(3) A Few Practice Deductive Arguments

(4) Testing for Invalidity

(5) Practice Exercises

# Deductive and Inductive Logic

# Deductive vs Inductive

## Deductive Reasoning

- Formal (the inference can be assessed from the form alone).
- When sound, the conclusion is *guaranteed* to be true.
- The conclusion **is extracted from** the premises.

## Inductive Reasoning

- Informal (the inference *cannot* be assessed by the form alone).
- When cogent, the conclusion is only probably true.
- The conclusion **projects beyond** the premises.

# Deductive Logic: Basic Terms

## Validity

- A property of the **form** of the argument.
- If an argument is valid, then the truth of the premises guarantees the truth of the conclusion.

## Soundness

- A property of the **entire argument**.
- If an argument is sound, then:
  (1) it is valid, and
  (2) all of its premises are true.

# Validity

If an argument is **valid**, then the truth of the premises guarantees the truth of the conclusion.

**A valid argument can have:**

- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

**A valid argument can not have:**

- True premises, false conclusion

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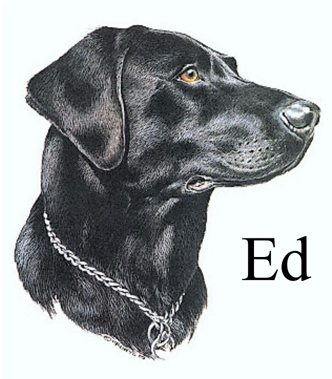
**A valid argument can not have:**

- True premises, false conclusion

All dogs are mammals.
Ed is a dog.
∴ Ed is a mammal.



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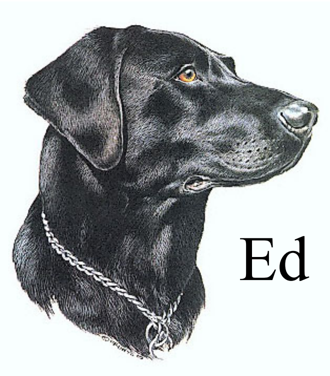
All cats are dogs.
Ed is a cat.
∴ Ed is a dog.



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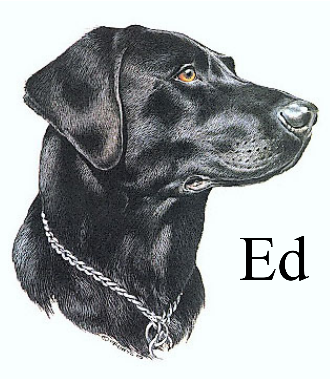
**A valid argument can have:**

- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

**A valid argument can not have:**

- True premises, false conclusion

All cats are toads.
Ed is a cat.
∴ Ed is a toad.



# Sample Deductive Arguments

# Deductive Argument #1

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s not raining.

∴ (3) You won’t need your umbrella.

# Checking for Invalidity

## Two Methods of Counter-example

### Alternate scenario

Imagine some **alternate scenario** in which the premises of the argument will be true, but the conclusion false.

### Substitution (two-step)

(1) Determine the form of the argument.

(2) Substitute other statements, such that all the premises will be true but the conclusion false.

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| | | |
|---|---|---|
| (1) | If R, then U | R = I'm a dog. |
| (2) | Not-R | U = I'm a mammal. |

∴ (3) Not-U

**[Denying the Antecedent]**

**INVALID**

# Deductive Argument #2

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s raining.

∴ (3) You’ll need your umbrella.

# Deductive Argument #2

(1) If it's raining, then you'll need your umbrella.

(2) It's raining.

∴ (3) You'll need your umbrella.

| | | |
|---|---|---|
| (1) | If R, then U | If P, then Q |
| (2) | R | P |
| ∴ (3) | U | ∴ Q |

**[Modus Ponens (Latin: "mode that affirms")]**

**VALID**

# Deductive Argument #3

If Ed has black hair, then Ed is Italian.

Ed does have black hair, so Ed is Italian.

# Deductive Argument #3

If Ed has black hair, then Ed is Italian.

Ed does have black hair, so Ed is Italian.

(1) If B, then I

(2) B

∴ (3) I

**[Modus Ponens]**

**VALID**

Ed

# Deductive Argument #4

If God exists, then there's no evil in the world. But there *is* evil in the world, so God must not exist.

# Deductive Argument #4

If God exists, then there's no evil in the world. But there *is* evil in the world, so God must not exist.

| | |
|---|---|
| (1) If G, then not-E | If P, then Q |
| (2) E | not-Q |
| ∴ (3) not-G | ∴ not-P |

**[Modus Tollens (Latin: "mode that denies")**

**VALID**

# Deductive Argument #5

If the medicine doesn’t work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.

# Deductive Argument #5

If the medicine doesn't work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.

| | |
| --- | --- |
| (1) If not-W, then D | If P, then Q |
| (2) D | Q |
| ∴ (3) not-W | ∴ P |

**[Affirming the Consequent]**

**INVALID**

# Deductive Argument #6

That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.

# Deductive Argument #6

That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.

| | |
|---|---|
| (1) J or M | P or Q |
| (2) not-J | not-P |
| ∴ (3) M | ∴ Q |

**[Disjunctive Syllogism]**

**VALID**

# Practice Arguments

# Practice Argument #1

If he was lost, then he would have asked for directions. But he didn't ask for directions. So he must not be lost.

| | |
|---|---|
| (1) If L, then D | If P, then Q |
| (2) not-D | not-Q |
| ∴ (3) not-L | ∴ not-P |

**[Modus tollens]**

**VALID**

# Practice Argument #2

If interest rates drop, then the dollar will weaken against the Euro. Interest rates did drop. Therefore, the dollar will weaken against the Euro.

| | | |
|---|---|---|
| (1) | If I, then D | If P, then Q |
| (2) | I | P |
| ∴ (3) | D | ∴ Q |

**[Modus ponens]**

**VALID**

# Practice Argument #3

If his light is on, then he's home. But his light isn't on, so he's not home.

| | | |
|---|---|---|
| (1) | If L, then H | If P, then Q |
| (2) | not-L | not-P |
| ∴ (3) | not-H | ∴ not-Q |

**[Denying the Antecedent]**

**INVALID**

# Practice Argument #4

The mind is an immaterial substance, for it is either identical to the brain or it is an immaterial substance, and it's not identical to the brain.

| | |
|---|---|
| (1) B or I | P or Q |
| (2) not-B | not-Q |
| ∴ (3) I | ∴ P |

**[Disjunctive Syllogism]**

**VALID**

# Practice Argument #5

If you want to get into law school, then you'd better do your logic homework.

| | | |
|---|---|---|
| (1) | If L, then H | If P, then Q |
| [(2) | L] | P |
| [∴(3) | H] | ∴ Q |

**[Enthymeme, expanded as modus ponens]**

**VALID**

# Practice Argument #6

If you're wealthy, then you've spent years and years in school.  Think about it: If you're a brain surgeon, then you're wealthy.  And if you're a brain surgeon, then you've spent years and years in school.

| | | |
|---|---|---|
| (1) | If BS, then W | If P, then Q |
| (2) | If BS, then S | If P, then R |
| ∴ (3) | If W, then S | ∴ If Q, then R |

**[fallacy]**

**INVALID**

# Determining Validity

## To determine invalidity…

… we can use the method of counter-example.

## To determine validity…

… we need something else: Truth Tables

# Truth Tables

Example

(1) If I win the lottery, then I'll buy you dinner.. If p, then q

(2) I won the lottery.. p

(3) I'll buy you dinner. ∴ q

| | p | q | if p, then q | p | q |
|---|---|---|---|---|---|
| 1 | T | T | T | T | T |
| 2 | T | F | F | T | F |
| 3 | F | T | T | F | T |
| 4 | F | F | T | F | F |
| | | | P1 | P2 | C |

indicates validity

Why do conditionals have these truth-values?

# Truth Tables

Example

(1) If it's raining, then you'll need your umbrella. If p, then q

(2) It's not raining. not-p

(3) You don't need your umbrella. ∴ not- q

| | p | q | if p, then q | –p | –q |
|---|---|---|---|---|---|
| 1 | T | T | T | F | F |
| 2 | T | F | F | F | T |
| 3 | F | T | T | T | F |
| 4 | F | F | T | T | T |
| | | | P1 | P2 | C |

indicates invalidity

# The TV of Conditionals

The logic of conditional statements is such that they are false only when the antecedent is true and the consequent is false.

A= **If I win the lottery, then I'll buy you dinner.**

Suppose…

(1) I both win the lottery and buy you dinner. (A is true)

(2) I win the lottery, but don't buy you dinner. (A is false)

(3) I lose the lottery, but still buy you dinner. (A is true)

(4) I lose the lottery, and don't buy you dinner. (A is true)

# “Or”

In **English**, ‘or’ can be used either inclusively or exclusively:

Inclusive “or”: “P or Q or both”

Example: “He’s either reading a book or out in the garden (or both).”

Exclusive “or”: “P or Q but not both”

Example: “The train’s coming in on either platform 3 or platform 5.”

In **logic**, “or” is always understood in the inclusive sense.