

Arithmetic for Computers

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Arithmetic at Hardware Level

- Representing negative numbers
- Fractions, real numbers
- Limits of representation
- Overflow issues
- Hardware multiplication and division

Representing Numbers

- 32-bit word
 - represents 2^{32} different bit patterns
 - Start from 0, till $2^{32} - 1$ ($4,294,967,295_{10}$)
 - $(b_{31} \times 2^{31}) + (b_{30} \times 2^{30}) + \dots + (b_1 \times 2^1) + (b_0 \times 2^0)$
- Conversion from binary to decimal and back
- Addition with overflow

Two's Complement for Signed Numbers

- *Observation* : -ve numbers from subtraction result in leading 1's
- Let: leading 1's $\rightarrow$ -ve; leading 0's $\rightarrow$ +ve
 - 0 to $2^{31} - 1$: same as before
 - $-2^{31} = 1000 \dots 000_{\text{two}}$, $-(2^{31} - 1) = 1000 \dots 001_{\text{two}}$
 - $111 \dots 110_{\text{two}} = -2$ and $111 \dots 111_{\text{two}} = -1$
- MSB: *sign bit*. If 1 $\Rightarrow$ -ve, if 0 $\Rightarrow$ +ve
- LSB: tells if odd or even
- *Shortcut*: to negate a number, x (+ve or -ve)

$\bar{x} \equiv$ logical complement of x

$$X_n = \bar{x} + 1$$