Arithmetic for Computers

October 31, 2008

ALU

- Arithmetic Logic Unit
- Performs most processor operations
- *Control* signal predicates
  - addition
  - subtraction
  - logic operations
  - shifting (w/ or w/o sign extension)

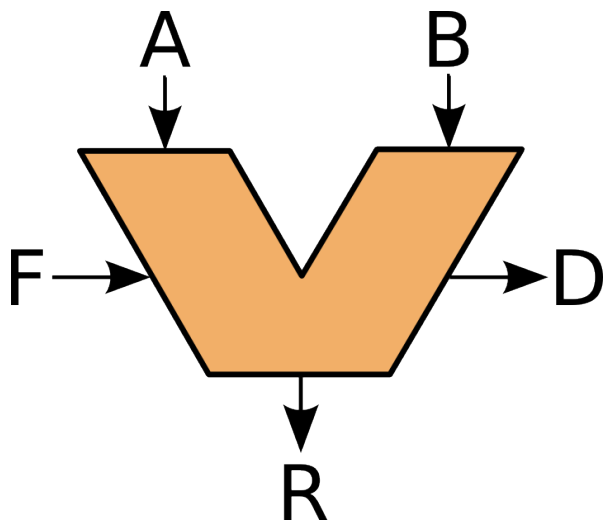

Figure: ALU symbol

Multiplication

- Product of `n` with `m` bits = `n` + `m` bits long
- More complicated than addition
  - use addition with shifting
  - shift left 1 bit to align with previous sum

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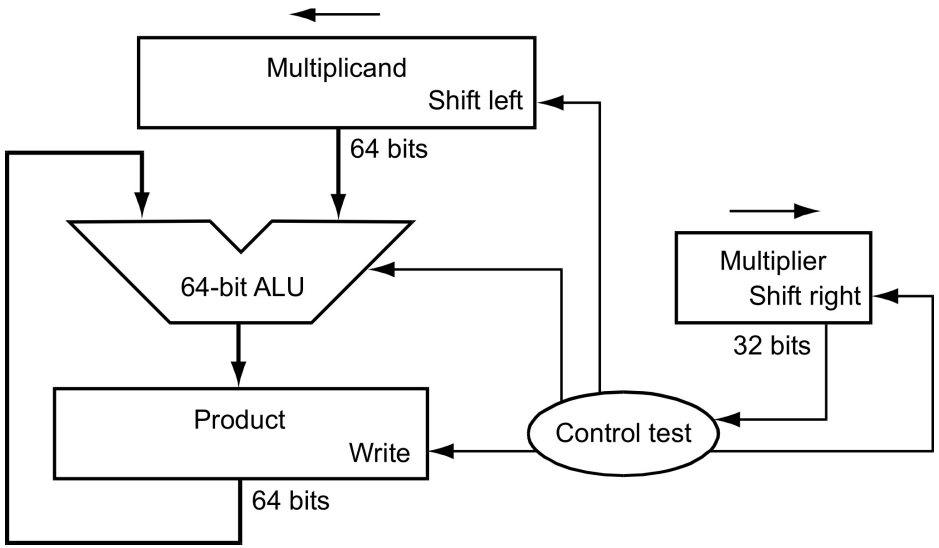
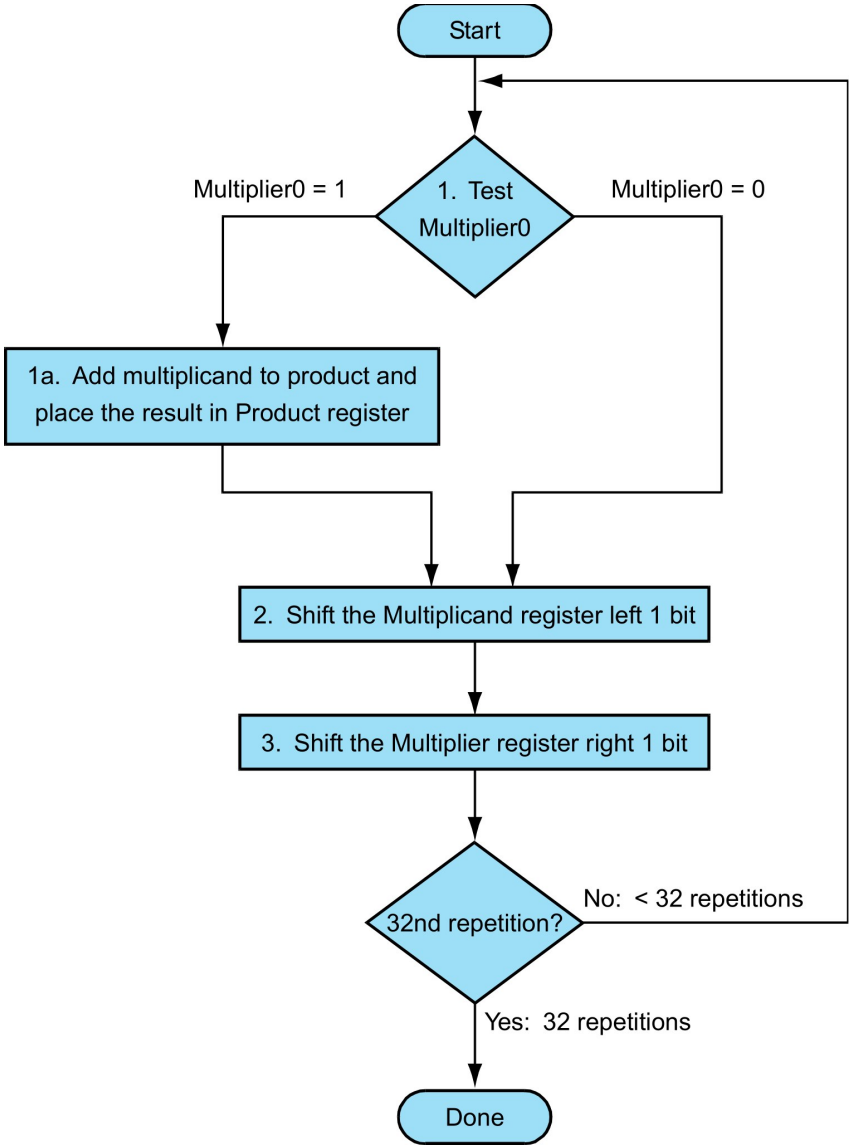


Figure: Hardware for multiplication

- 64-bit multiplicand reg.: ∵ will be moved 32 times
  - 32-bit multiplicand in right half; left half: 0‘s
- Product: initialized to 0‘s

Multiplication Algorithm



Improvement

- Problem: need 64-bit adder
- Half bits in multiplicand are 0
- Improvement:
  - Don't shift the multiplicand
    - Shift *product right*!
  - Add it to product on the *left*
  - Shift product to *right*

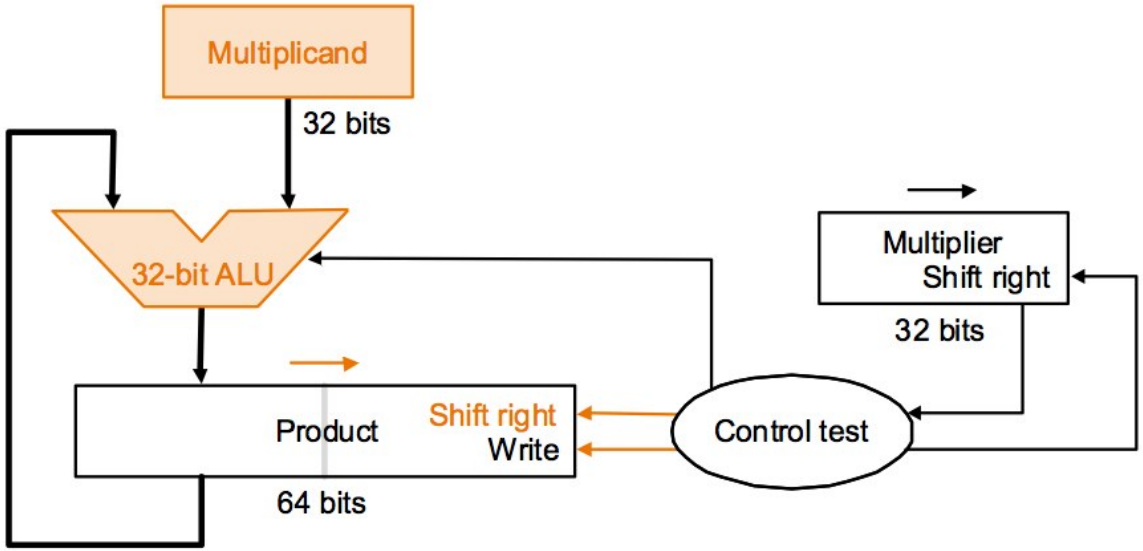
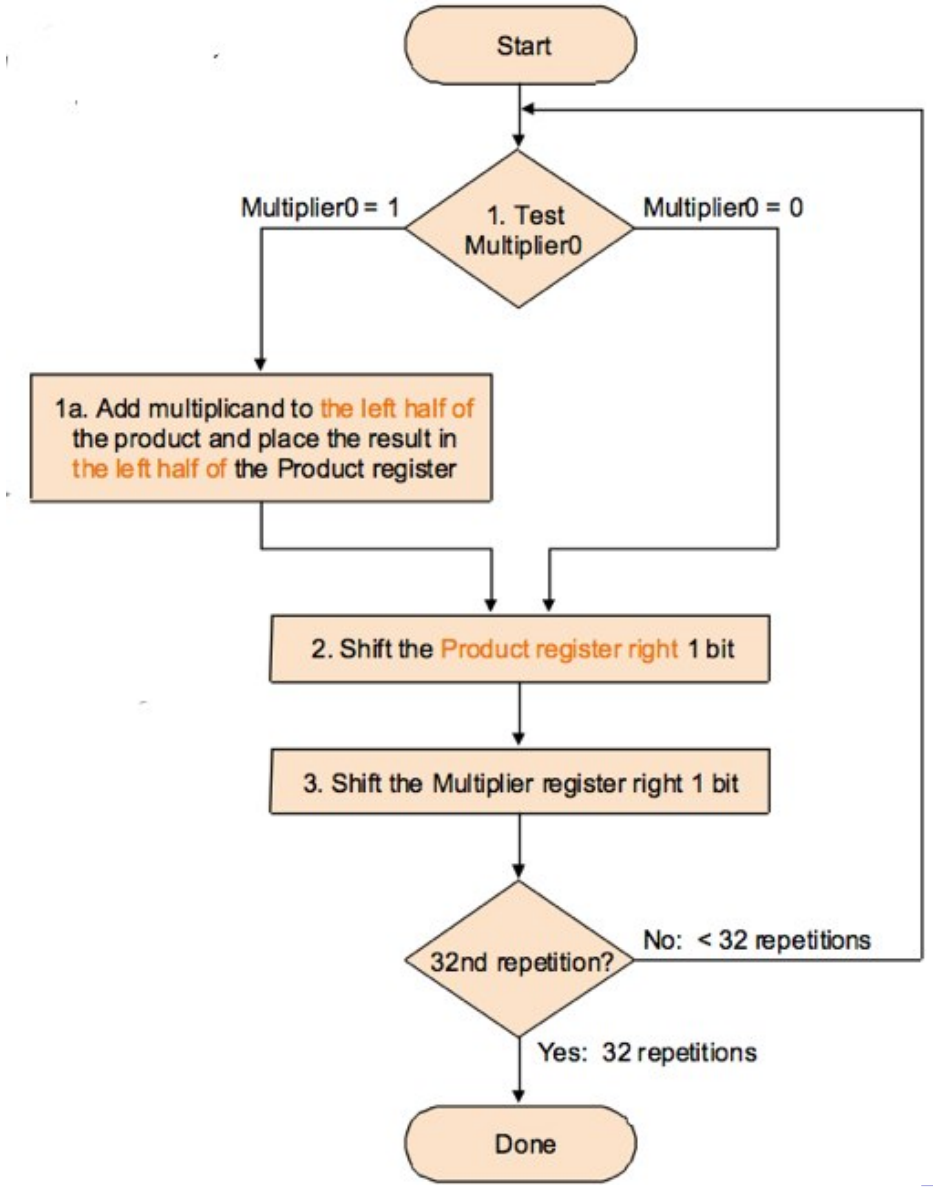


Figure: Improved hardware

New Algorithm



Another Improvement

- Save on space:
  - Put multiplier in product
  - saves on speed: only single shift needed

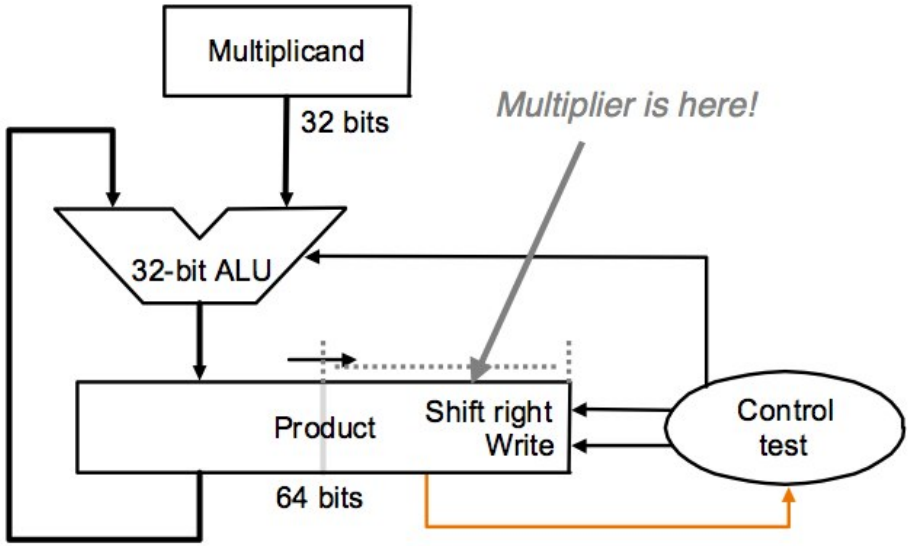


Figure: Improved hardware for multiplication