MATH 103 - Test #1 - 10/8/98

Show all work for full credit. Points in [brackets] total 100.

1. Suppose City A begins with 100,000 people in 1990 and grows by 5000 people per year. City B has 50,000 people in 1990 and then grows 5% per year.

a. Write an equation for each city describing the population *P* after time *t* (t = years after 1990).

[6]

City A: P =

City B: P =

b. By how many people does each city grow during the *first* year?

[5]

City A

City B

c. Explain in two or three sentences why City B's population will eventually overtake City A, assuming the above growth patterns continue indefinitely. (Your answer should relate to the number of people added each year.)

[4]

2. a. A quantity which grows by the same *amount* every time period is said to be

[3] experiencing (exponential / logistic / linear) growth. **Circle correct word.**

b. A quantity which grows by the same *percent* every time period is said to be

[3] experiencing (exponential / logistic / linear) growth.

c. Make rough sketches of functions which correspond to the situations described in #2a and #2b.

[4]

(a) (b)

3. The following table summarizes the projected growth of two towns.

	1990 Pop.	Annual Growth
Town A	5000	5%
Town B	1000	10%

a. In the long-run, which town will have the larger population? **Explain your answer in one or two full sentences.**

[4]

b. Write a function describing the population *P* as a function of time *t* (t = years after 1990).

[6]

Town A: P =

Town B: P =

c. Determine the population of each town in the following years:

[6]

	A	B
1990		
2010		
2030		

d. Using the information in (b) draw your two functions on the same set of axes.

[6]

e. Use the graph in (c) to estimate when Town B's population will catch up with Town A's.

[4]

4. The population of the United States was 205 million in 1970. The birth rate was 18.2 per thousand people and the death rate was 9.5 per thousand people.

a. Using the "Malthusian Growth Model", write a function describing the population *P* as function time *t* (t = years after 1970). (If you don't recall the Malthusian model then you can use another reasonable equation for two-thirds credit!)

[5]

P =

b. Use your function from (a) to estimate the U.S. population in the year 2070.

[5]

c. Your answer to (b) makes a critical assumption. State this assumption and discuss whether you believe it is very realistic.

[4]

5. Sometimes people use the phrase "exponential" to mean a quantity is growing very quickly (e.g. "crime is out of hand -- it is growing exponentially "). Critique this usage of the term "exponential" (Is it an appropriate usage -- why or why not?)

[5]

6. Consider the following linear program (LP):

minimize	C =	3x + 2y
subject to		2x + y $10
		4x + 3y $24
		x$0, y$0

a. Sketch a graph of the *feasible region* for this LP. **Show all calculations required to determine the corner points of this region.**

[12]

b. Determine the minimum value of *C* for this linear program. **Show all work!**

[5]

c. What is the *optimal solution* for this linear program?

[2]

7. Your company makes two products -- tables and chairs. The profit per item and

[10] resources (wood, metal and time) required to build each item are given below.

	Tables	Chairs
Profit ($)	20	10
Wood (lbs.)	10	5
Metal (lbs.)	8	3
Time (hrs.)	12	6

In a given week you have available 200 lbs. of wood, 100 lbs. of metal and 120 hours of labor. Write a *linear program* which could be used to determine how many of each item should be built per week to *maximize the total profit*. Your answer should be complete in all respects. **(NOTE: You do not need to solve the linear program -- only set it up! Answer on the back of the previous page.)**