

Math 103 - Test #1 - 10/8/99

Points in [brackets] total 100. Show all work for full credit!

1. Consider Countries A, B and C with the following characteristics.
[35]

Current Population

Projected Growth

A)	20 Million	3 percent per year
B)	15 Million	5 percent per year
C)	25 Million	0.1 million people per year

- a. For each of these countries **write an equation** which describes the population P as a function of time t . ($t =$ years from now) and **calculate the country's population** 5 years from now.
- A)
- B)
- C)
- b. Assuming that these patterns continue indefinitely into the future,
- which population will be the *largest*, "in the long run?" A. B. C.
- which population will be the *smallest*, "in the long run?" A. B. C.
- c. Explain your answers to (b) based on the *type of growth* that is occurring and the *rates* involved.

2. A population of elephants in a particular African reserve currently contains 10,000 members. We observe that the annual birth rate is 30 per 1000 elephants and the death rate is 20 per 1000. Using the *Malthusian growth model*,

[20]

- a. Write an equation describing the population P as a function of time in years t .
- b. Determine the projected size of the population 10 years from now.
- c. Is it reasonable to believe that the equation in (a) will provide a realistic projection over the next 100 years? **In a complete paragraph, explain why or why not.**

3. Suppose you place \$1000 in a savings account and each month you are paid 1% interest. You leave all of the initial principal (the \$1000) and the added interest in your account and do not make any further deposits.

[10]

- a. The first month the bank will add \$10 interest (1% of 1000) to your account. What will happen to the *amount* of interest they put in your account in each successive month (will it increase, decrease, or remain the same)? **Explain your answer fully.**
- b. Explain what is meant by the “compounding effect” in the context of this problem.

1. a. Suppose the *slope* of a linear growth function is 100. What does this tell you about how quickly the quantity is growing?

[10]

- b. Consider the equation $y = 50 \cdot 3^{1.4x}$. The graph of this function crosses the vertical axis at the point _____. This equation represents a quantity that is _____ (linear / constant / increasing / decreasing). **Circle one choice.**

5. Suppose your company produces two types of bikes — mountain bikes and touring bikes. A

mountain bike requires 2 hours to manufacture and 2 hours to assemble, while a touring bike requires 2 hours to manufacture and 3 hours to assemble. Each mountain bike yields \$70 profit while each touring bike yields \$100 profit. Your company has 40 hours per week available for manufacturing and 42 hours per week for assembly. You wish to determine how many of each bike should be made to maximize the total profit per week.

[25]

- a. Set up this problem as a linear program (LP) by filling in the objective function and the assembly constraint. (Let x = mountain bikes, y = touring bikes)

Objective function: $P =$ _____

Constraints: $2x + 2y \leq 40$ (Manufacturing)

_____ (Assembly)

$x \geq 0, y \geq 0$

- b. Graph the feasible region for this LP.

- c. Use the “corner principle” to determine how many of each bike should be produced.

Mountain Bikes =

Touring Bikes =

- d. What is the maximum possible profit?