MATH 103 - Final Exam - Spring 2000

Points sum to 150. Show all work or maximum credit!

1. "A Tale of Two Cities"
[20]

- City A currently has 50,000 people and is projected to grow 5,000 people per year.
- City B currently has 40,000 people and is projected to grow 10% per year.

a. Complete the following table, indicating the population for Cities A and B over the next five years.

Year	City A	City B
2000	50,000	40,000
2001		
2002		
2003		
2004		
2005		

b. Which of these cities is undergoing *linear* growth? Which is *exponential* growth? **Justify your answers.**

c. If these growth patterns continue indefinitely which city will have the larger population "in the long run." **Make sketches of the two growth patterns to support your answer.**

2. You wish to estimate the probability that your roommate Fred attends his 8 a.m. class. You randomly pick 20 class days and observe that he attended 16 of these days.

[15]

a. What is your best estimate for the probability of Fred attending?

b. Is the value you computed in (a) an "empirical" probability or a "theoretical" probability? **Explain!**

c. Based on your result in (a), how many times do you expect Fred to attend over the entire semester, consisting of 52 days?

3. In 1996, the U.S. government reported that the *mean* annual income for people of working age was $25,466. They also reported that the *median* income was $17,587. (This data includes those working part-time or not employed at all.)

[15]

a. Explain why the mean is generally higher than the median when describing income data.

b. Which better represents the typical income of people in the U.S. — the mean or the median? **Justify your answer and include reference to a histogram which would represent this data.**

c. You work for the IRS and wish to estimate the total tax revenue the government will receive from income taxes. Which would be more useful for you to know — the mean or the median income? **Explain!**

4. On a roulette wheel there are 38 slots. The player chooses one of the slots, places a bet and wins if the ball lands in the selected slot.

[15]

a. Assuming neither the casino nor you can control where the ball lands, what is the probability that you win?

b. You walk up to the roulette table and win on your first three plays. Your friends who are with you make the following comments:

- "Lady Luck is on your side — better keep playing."
- "Winning four times in a row is almost impossible — you better stop now."

Critique the comments of your friends. What does each of these comments assume regarding the chances of winning on the next play? Are these valid assumptions?

5. Students in my statistics class found that the amount of sleep that Manchester students get on week nights is normally distributed with a mean of 6.5 hours and standard deviation of .5 hours. State a range that contains the sleep times for almost all (close to 100%) of M.C. students.

[10]

6. You are planning a Memorial Day picnic for your family. You must decide on the location one month in advance. You have narrowed it down to three choices: a) the beach on Lake Michigan, b) a state park, or c) your house. The beach has no sheltered picnic areas, while the park has a nice covered pavilion. Your house doesn't have much for outdoor activities but it is air-conditioned. (If you have it at your house and its is a nice day, however, everyone will complain that you should have had the picnic elsewhere.) The park tends to have a lot of mosquitoes on damp days.

[25]

In making your decision you want to take into account these factors and also the possible weather conditions. The following table summarizes the pleasure you and your guests will receive from the picnic depending on the location and the weather.

	Cool & Cloudy	**Warm & Clear**	**Sweltering Hot**	**Thunder-storms**
a) Beach	5	10	7	!3
b) Park	4	8	5	3
c) House	6	4	7	6

Use each of the five decision-making criteria to make your decision. For each method, be sure to *indicate the benefit associated with each of the three picnic locations and clearly state the alternative chosen.*

a. Criterion of Certainty (based on past observation it is usually warm and clear on Memorial Day — assume this will occur this year and make your decision.)

< Beach
< Park
< House

b. Criterion of Pessimism

< Beach
< Park
< House

c. Criterion of Optimism

< Beach
< Park
< House

d. Criterion of Expected Value (Historical data provide the following probabilities.)

Weather	Probability
Cool and Cloudy	.15
Warm and Clear	.60
Sweltering Hot	.20
Thunderstorms	.05

< Beach

< Park

< House

e. Criterion of Equal Probability

< Beach

< Park

< House

7. Consider the following game. On each play of the game each player must choose either RED or BLUE. The pair (x, y) indicates that Player #1 receives *x* points and Player #2 receives *y* points. The object is for each player to maximize their own point total.

[25]

Player #1 chooses:	Player #2 chooses:	
	RED	BLUE
RED	(2, 2)	(5, 1)
BLUE	(1, 5)	(4, 4)

a. Explain why choosing BLUE is considered to be "cooperating" with the other player.

b. Suppose both players just chose BLUE. Would Player #1 be tempted to switch to RED on the next round? **Explain!**

c. If Player #1 chose RED for several rounds, what would Player #2 likely do? **Explain!**

d. Based on (c) why is switching from BLUE to RED probably *not* in Player #1's *long-term interest*?

e. Identify the "equilibrium point" in this particular game. Explain how this affects the game in terms of the players' ability to cooperate.

f. Explain (on the back side) the "tit-for-tat" strategy for playing this game and indicate why this strategy can be described as "kind," "provokable," "forgiving," and "clear."

8. [25] For each definition find the lettered item on the next page to which it corresponds. (It is probably most efficient to skim through the list of terms and then work through the list of definitions.) **Each definition should have *exactly one* term assigned to it. Not all lettered items in the terminology list will be used. No term is to be used more than once.**

___ 1. Repeated increase by the same percent in each time period.

___ 2. Repeated increase by the same amount in each time period.

___ 3. The rate of change in a linear function.

___ 4. A histogram in which one side is the mirror image of the other.

___ 5. Monetary values (wealth, income, price) follow this pattern since there are no upper limits.

___ 6. Assume that, in the future, the worst case will always occur.

___ 7. The results from a survey tend to always be either too high or too low.

___ 8. Probabilities for future conditions are known.

___ 9. The proportion of time that an outcome occurs in a small experiment.

___ 10. The proportion of time that an outcome occurred in. the long-run.

___ 11. The ratio of times an event occurs to times it does not occur.

___ 12. The number of values in a set of data that fall in a certain range.

____ 13. A model often used to represent physical measurements, such as height, weight, aptitude.

___ 14. The average outcome for a random phenomenon, over the long-run.

___ 15. A measure of how much a set of data varies from the mean.

___ 16. The "average" obtained by adding the data values and dividing by the number of values.

___ 17. The "middle" value in a set of ordered data.

___ 18. In a normal distribution, the number of standard deviations a value is from the mean.

___ 19. Arrangements (or orderings) of a set of objects.

___ 20. The number of ways you can choose a given number of objects from a larger group.

___ 21. A usually large group about which you are trying to reach a conclusion.

___ 22. A small group from which data is actually collected.

____ 23. A number describing the accuracy of an estimate obtained from a survey.

____ 24. Neither player has an incentive to unilaterally switch their choice.

___ 25. All outcomes of a random experiment are equally likely to occur.

Terms for Matching

a. linear growth
b. exponential growth
c. slope (or rate of change)
d. compound interest
e. relative frequency
f. probability (i.e., theoretical)
g. empirical probability
h. “fair”experiment
i. odds
j. expected value
k. factorial
l. permutation
m. combination
n. “Basic Counting Law”
o. frequency
p. histogram
q. median
r. mean
s. standard deviation
t. normal distribution
u. z-value
v. symmetric
w. skewed
x. “68-95-99.7 Rule”
y. population
z. sampling frame
aa. sample
bb. sampling bias
cc. confidence interval
dd. margin of error
ee. confidence level
ff. decision-making under certainty
gg. decision-making under risk
hh. criterion of pessimism
ii. criterion of optimism
jj. criterion of equal probability
kk. equilibrium point (of a game)
ll. cooperation
mm. defection
nn. majority
oo. plurality

EXTRA CREDIT [Up to 15 extra points]

There are three candidates running for Student Senate President: George Jetson, Betty Rubble and Magilla Gorilla. The voters are asked to place the candidates in order of preference. The results are summarized below. For example, 40% of the voters selected G. Jetson as their first choice, B. Rubble as their second choice and M. Gorilla as their third choice.

	40%	35%	25%
Jetson	1	3	3
Rubble	2	1	2
Gorilla	3	2	1

4. Suppose we are use plurality voting.

   a. Who would be declared the winner? **Explain why.**

   b. What is the weakness in using this method. (Use this election as an example.)

2. Suppose we use majority voting.

   a. Why is their no clear majority from the above results?

   b. What would need to done next to obtain a majority?

   c. Who would be the final winner? **Explain why!**

3. Suppose we use the Borda count ("rank-and score") method. Who would be declared the winner? **Using the back of this sheet, show all calculations needed to reach your conclusion.**