PHYS 485 Materials Physics **Spring 2019**

Homework 05

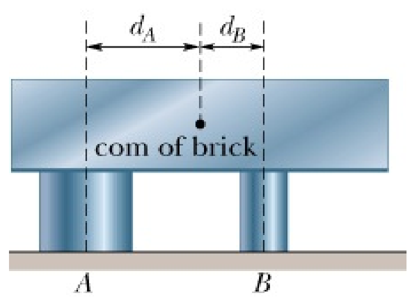
Read: Ch 3 (Stokes) and Ch 15, Section 6 of Knight's **Physics for Scientists and Engineers** (3rd ed), and Ch 4 of Holgate's **Understanding Solid State Physics** (this is the book I put in the electronics lab for you all to share)

Due date: 15 Mar 2019

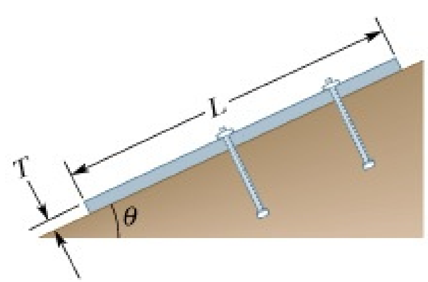
Work problems 2.22, 2.23, 3.1, 3.2 in the text.

AQ 1: First, a few warm-ups with stress and strain!
(a) A 70 kg mountain climber dangling in a crevasse stretches a 50 m long, 1.0 cm diameter rope by 8.0 cm. What is Young's modulus for the rope?
(b) A 3.0 m tall, 50 cm concrete column supports a 200,000 kg load. By how much is the column compressed? ($Y_{concrete} = 3 \times 10^{10}$ n/m^2)

AQ 2: In the figure below, a lead brick rests horizontally on cylinders A and B. The areas of the top faces of the cylinders are related by $A_A = 2\ A_B$. The Young's moduli of the cylinders are related by $Y_A = 2\ Y_B$. The cylinders had identical lengths before the brick was placed on them. What fraction of the brick's mass is supported (a) by cylinder A and (b) by cylinder B? The horizontal distance between the center of mass of the brick and the centerlines of the cylinders are d_A and d_B for cylinders A and B, respectively. (c) What is the ratio d_A/d_B?



AQ 3: In the figure below, a rectangular slab rests on a bedrock surface inclined at an angle $\theta = 26^o$. The slab has a length L = 43 m, thickness T = 2.5 m, and width W = 12 m. The density of the slab is 3.2 g/cm^3. The coefficient of static friction between the slab and the bedrock is 0.39.
(a) Calculate the component of the gravitational force on the slab parallel to the bedrock surface.
(b) Calculate the magnitude of the static frictional force on the slab. My comparing parts (a) and (b), you can see that the slab is in danger of sliding. This is prevented only by chance protrusions of bedrock. (c) To stabilize the slab, bolts are to be driven perpendicular to the bedrock surface (two bolts are shown). If each bolt has a cross-sectional area of 6.4 cm^2 and will snap under a shearing stress of 3.6×10^8 N, what is the minimum number of bolts needed? Assume that the bolts do not affect the normal force.



AQ 4: A. With a very simple approximation, we can relate the effect of a force F on a one-dimensional chain of atoms to the spring constant, α. Show that α is given by

$$\alpha \approx Y a_o$$

where Y is Young's modulus and a_o represents the width/height of a "bar" of the material that is constructed from the single chain (*i.e.,* the lattice parameter).

B. Find values for the spring constant, α, and the associated frequencies of vibration, ω, for diamond, copper, and lead using the following data:

Diamond: $Y = 9.5 \times 10^{11}\ N/m^2$, $a_o = 0.154\ nm$, $m = 12\ amu$
Copper: $Y = 1.3 \times 10^{11}\ N/m^2$, $a_o = 0.256\ nm$, $m = 64\ amu$
Lead: $Y = 1.5 \times 10^{11}\ N/m^2$, $a_o = 0.350\ nm$, $m = 207\ amu$

AQ 5: Show that the exponential formalism for the Fourier series given by

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega t}, \quad n = 0, \pm 1, \pm 2, \ldots$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-in\omega t}\, dt$$

is equivalent to the formalism that we learned in Analytical Mechanics, namely that

$$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

where

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega t)\, dt, \quad n = 0, 1, 2, \ldots$$

and

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega t)\, dt, \quad n = 1, 2, 3, \ldots$$