Lesson: Permutations and Combinations
Length: 45 minutes
Age/Grade Intended: Algebra II

Academic Standard(s):
A2.8.4
Use permutations, combinations, and other counting methods to determine the
number of ways that events can occur and to calculate probabilities, including the
probability of compound events.

Performance Objectives:
Students will solve word problems using permutations, combinations, and other
counting techniques when given a worksheet with 85 percent accuracy.

Assessment:
Students will be given a worksheet with 15 counting problems. They will be
grade on the accuracy of their work. The worksheet is due at the beginning of class the
next day.

Advanced Preparation by Teacher:
Materials -
Lunch menu for today
The Sundae Scoop by Stuart J. Murphy
Dry erase board
Dry erase markers

Preparation -
A copy of the children’s book The Sundae Scoop by Stuart J. Murphy, and
today’s lunch menu from the cafeteria will need to be acquired before the lesson.
Also, make sure there are enough copies of the “Count Me If You Can!”
worksheet for each student.

Procedure:
Introduction -
Begin the lesson by explaining to the students that they will be learning
different counting techniques such are permutations and combinations. Read
aloud to the class The Sundae Scoop. It is a children’s book that illustrates the
mathematical concept of combinations.
After reading the story, ask the students what is a combination. (Bloom:
Level I - Knowledge) Then ask them how they think combinations apply to probability and statistics. (Bloom: Level III - Application) Explain that combinations are a way of counting possible events. They will now learn two other counting techniques.

Plan -

1. Ask the students to get out their Math Journals. They will be adding to the permutation and combination entries. They may also want a notebook to take additional notes.

2. On the board, write: Fundamental Counting Principle (FCP)
   - Explain that the FCP is a way to count using multiplication.
   - If event M can occur in m ways followed by event N that can occur in n ways, the event M followed by N can occur in $m \times n$ ways.

3. Examples for FCP: (These should also be written on the board.)

   **Example 1** - How many different outfits can you wear if you have 3 pants and 2 shirts to choose from.
   - One way to solve is to make a tree diagram:
     
     Pants 1
     
     Shirt 1
     
     Shirt 2
     
     Pants 2
     
     Shirt 2
     
     Shirt 1
     
     Pants 3
     
     Shirt 2
     
   *There are 6 outfits to choose from
   - Or you can use FCP: $3 \times 2 = 6$ outfits.

   **Example 2** - Using today’s lunch menu, determine how many different meals there are to choose from by multiplying the number of sandwiches, sides, desserts, and drink options.

   - Explain that permutations count the number of ways to order r objects selected from a set of n objects.
   - The equation is $nPr = n!/(n-r)!$
   - Explain that $n!$ means “n-factorial” and that $n! = n(n-1)(n-2) \ldots 1$
4. Examples for permutations:

   **Example 1** - In how many different ways can you file 12 folders, one after another, in a drawer?
   - Use FCP to count # of permutations
     \[ 12 \times 11 \times \ldots \times 1 = 12! = 479,001,600 \]
   - There are 479,001,600 ways to file 12 folders in a drawer.

   **Example 2** - Ten students are in a race. First, second, and third places will win medals. In how many ways can 10 runners finish first, second, and third.
   - One way to solve is to use the FCP:
     \[ 10 \times 9 \times 8 = 720 \]
   - Or you can use the formula for nPr:
     \[ 10P3 = 10!/(10-3)! = 10!/7! = 720 \]

5. On the board, write: Combinations - treat the same.
   - Explain that a combination is the number of ways you can choose r objects form a group of n objects.
   - The equation is \( nCr = n!/r!(n-r)! \)

6. Examples for combinations:

   **Example 1** - What is the number of combinations of 13 items taken 4 at a time?
   - Use the formula to solve.
     \[ 13C4 = 13!/4!(13-4)! = 13!/4!9! = 715 \]

7. Deciding whether to use permutations or combinations.
   - You have to ask does order matter? If it does, use permutations.
     If order does not matter, use combinations.
   - Read the following scenarios to the students and have them tell if you should use permutations or combinations (Bloom: Level II - Comprehension):
     **Scenario A** - A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawing to display. In how many different ways can he select the drawings?
     * Combination
     **Scenario B** - You will draw winners form a total of 25 tickets in a raffle. The first ticket wins $100. The second ticket wins $50. The third ticket wins $10. In how many different ways can you draw the three winning tickets?
     * Permutation
**Closure -**

Have the students get into groups of three to create their own permutation and combination scenarios (Bloom: Level V - Synthesis)(Gardner: Interpersonal).

- They should create one permutation scenario and one combination.
- Have the groups write down their ideas.
- Walk around to help the groups that need it.

Once the groups have finished, allow them to share their scenarios with the class.

After they have finished sharing their scenarios, pass out the homework worksheet titled “Count Me If You Can!” (Gardner: Logical-Mathematical) They may work for the remaining time. Walk around to help any student who may need it. Also, be sure to remind them that their entries for permutations and combinations should be filled out in their Math Journals.

**Adaptations/Enrichment:**

**ADD -** For students with ADD the lecture time may not be engaging enough. This lesson can be modified to better engage students with this disability in the following way. Physical manipulatives or online manipulatives can be used during the examples. For Permutations Example 2, the ADD student along with 9 other classmates can be used to demonstrate the example (Gardner: Kinesthetic and Visual). This will help break up the lesson to help keep the ADD student focused.

**Self-Reflection:**

Were there enough examples? Did I pick good examples to illustrate each concept? Was there enough time for this lesson? Was the read aloud book engaging? Do the students understand the difference between permutation and combination?
Name: __________________________

Count Me If You Can!

Solve use the correct formulas.
1. 12!/6!
2. 8P1
3. 9P6
4. 4C3
5. 7C7
6. 7C4/9C4

7. Use the definition of permutation to show why 0! should equal 1.

Tell whether it is a permutation or a combination. Do not solve.
8. How many different teams of 11 players can be chosen from a soccer team of 16?

9. Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?

10. A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?

Solve.
11. Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes on student. In how many ways can ten time slots be assigned?

12. There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?

13. In how many different ways can you arrange nine CDs one after another on a shelf?

14. An old web-site requires a four-character password consisting of three numbers and one letter. A new website requires a six-character password consisting of three numbers and three letters. How many more passwords can be made for the new website?

15. A consumer magazine rates televisions by identifying two levels of price, five levels of repair frequency, three levels of features, and two levels of picture quality. How many different ratings are possible?
Count Me If You Can!

*KEY*

Solve use the correct formulas.

1. \( 12! / 6! \)  
   \[ 12 \times 11 \times 10 \times 9 \times 8 \times 7 = 665,280 \]

2. \( 8P1 \)  
   \[ 8 \]

3. \( 9P6 \)  
   \[ 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60,480 \]

4. \( 4C3 \)  
   \[ \binom{4}{3} = \frac{4!}{3!(4-3)!} = 4 \]

5. \( 7C7 \)  
   \[ \binom{7}{7} = \frac{7!}{7!(7-7)!} = 1 \]

6. \( 7C4 / 9C4 \)  
   \[ \frac{\binom{7}{4}}{\binom{9}{4}} = \frac{\frac{7!}{4!(7-4)!}}{\frac{9!}{4!(9-4)!}} = \frac{35}{126} = \frac{5}{18} \]

7. Use the definition of permutation to show why 0! should equal 1.
   \[ nP0 = n!(n-0)! = n!/n! = 1 \]

Tell whether it is a permutation or a combination. Do not solve.

8. How many different teams of 11 players can be chosen from a soccer team of 16?  
   **Combination**

9. Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?  
   **Combination**

10. A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?  
    **Combination**

Solve.

11. Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes one student. In how many ways can ten time slots be assigned?  
    \[ 10,897,286,400 \]

12. There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?  
    \[ 56 \]

13. In how many different ways can you arrange nine CDs one after another on a shelf?  
    \[ 362,880 \]

14. An old web-site requires a four-character password consisting of three numbers and one letter. A new website requires a six-character password consisting of three numbers and three letters. How many more passwords can be made for the new website?  
    \[ 17,500,000 \]

15. A consumer magazine rates televisions by identifying two levels of price, five levels of repair frequency, three levels of features, and two levels of picture quality. How many different ratings are possible?  
    \[ 60 \]