Section 4.4

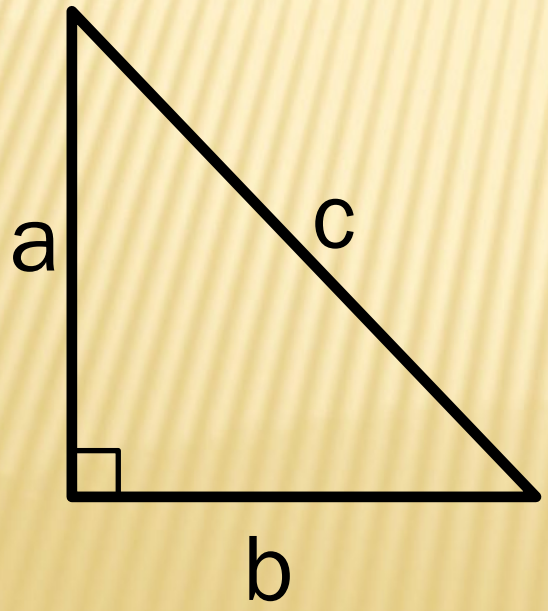
THE PYTHAGOREAN THEOREM AND THE DISTANCE FORMULA

VOCABULARY

Terms you need to know to understand this section

VOCABULARY

- The sides that form the right angle of a right triangle are called the legs.
- The side opposite the right angle is called the hypotenuse.



Which sides of this triangle are the legs and which side is the hypotenuse?

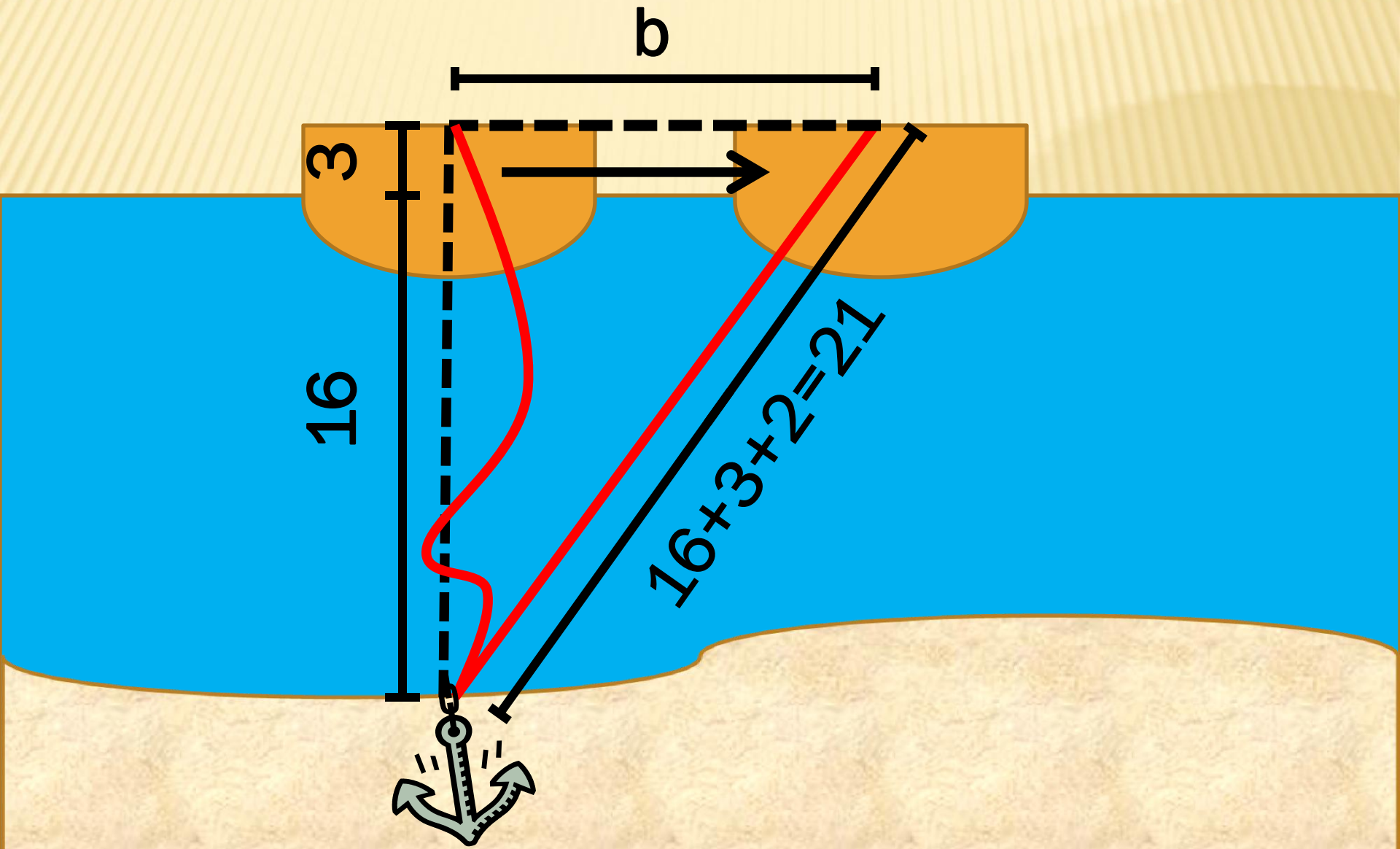
THE PYTHAGOREAN THEOREM

Problem statement, activity, and theorem

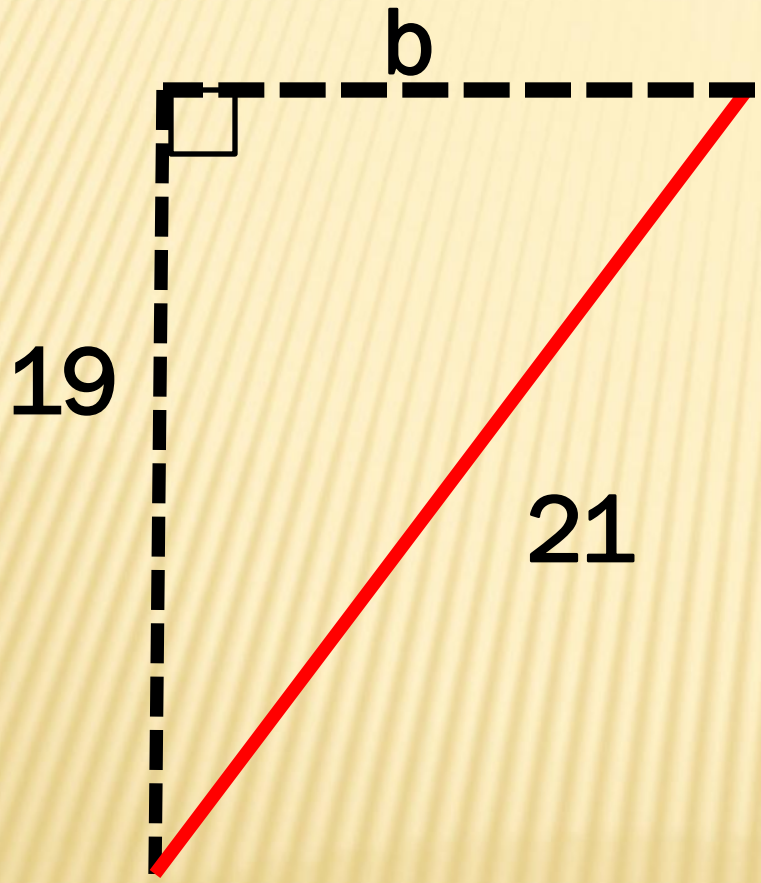
PROBLEM STATEMENT

When dropping an anchor, you are supposed to allow a little slack in the chain once you feel the anchor hit the bottom. Your dad tells you to drop the anchor over the side of the boat when his depth finder says the lake is 16 feet deep. You estimate that the side of the boat is 3 feet above the lake surface and you let out 2 extra feet of slack. Later, the chain is pulled tight because you have drifted. How far away is your boat from where you originally dropped the anchor?

DRAW A PICTURE



WITHOUT THE BACKGROUND



We need to determine the value of b. We can actually use what we are about to learn in order to solve this problem.

ACTIVITY

What you need:

- Pencil
- 1 piece of graph paper
- A ruler or straightedge

Directions:

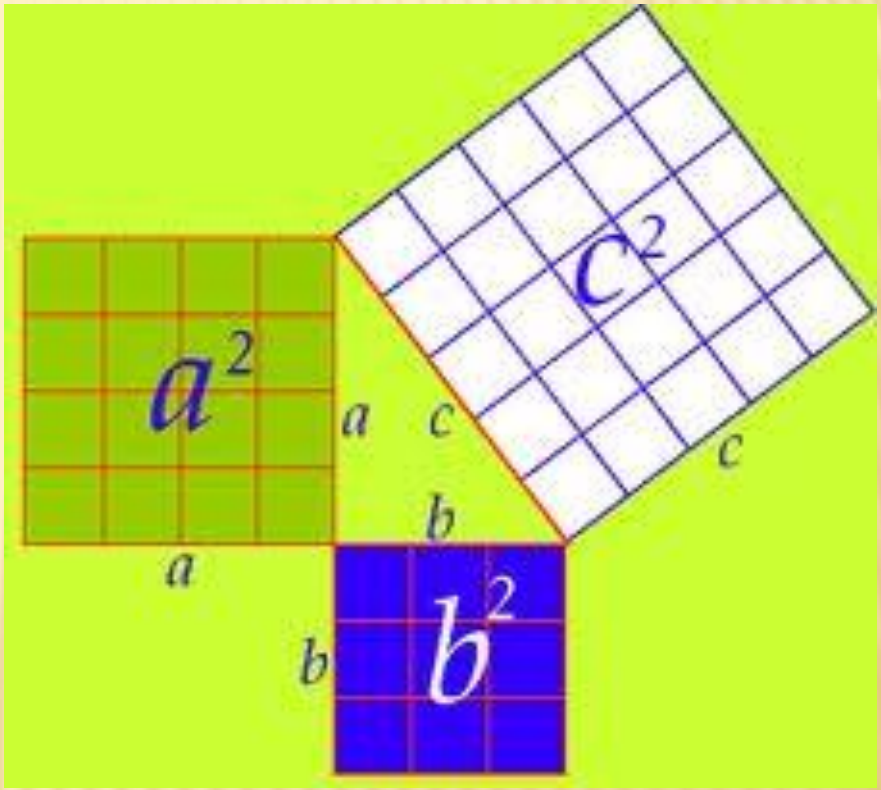
- On a piece of graph paper, draw a right triangle with legs that are three units each
- Draw a square from each side of the triangle.
- Find the area of each square by counting each grid.
- Repeat the process with a right triangle of a different size.

Can you find a relationship between the area of the squares?

RESULTS:

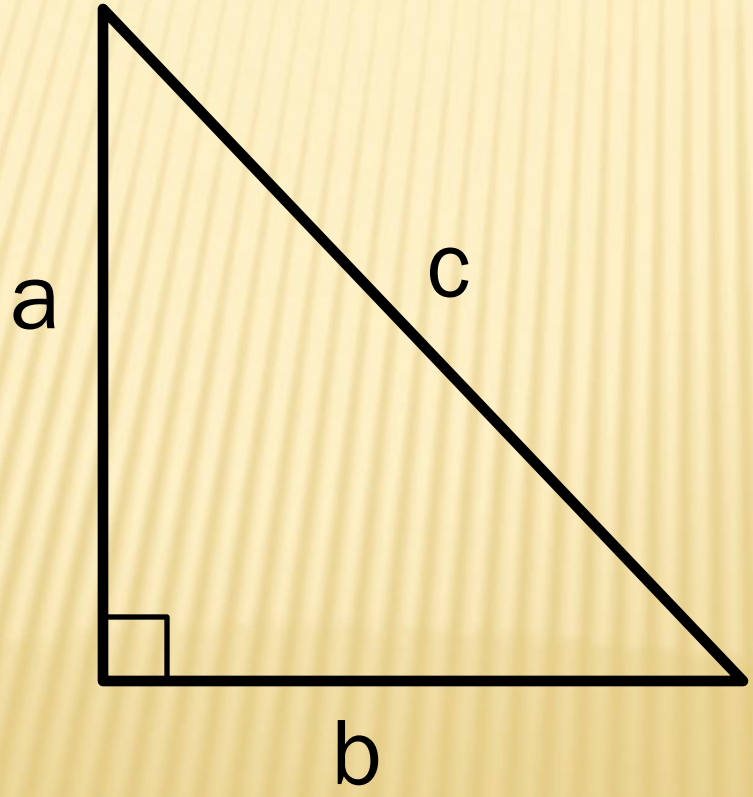
- The sum of the size of the squares of the legs is equal to the size of the square of the hypotenuse.
- The relationship can be described as

$$a^2 + b^2 = c^2$$



PYTHAGOREAN THEOREM

For a right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.

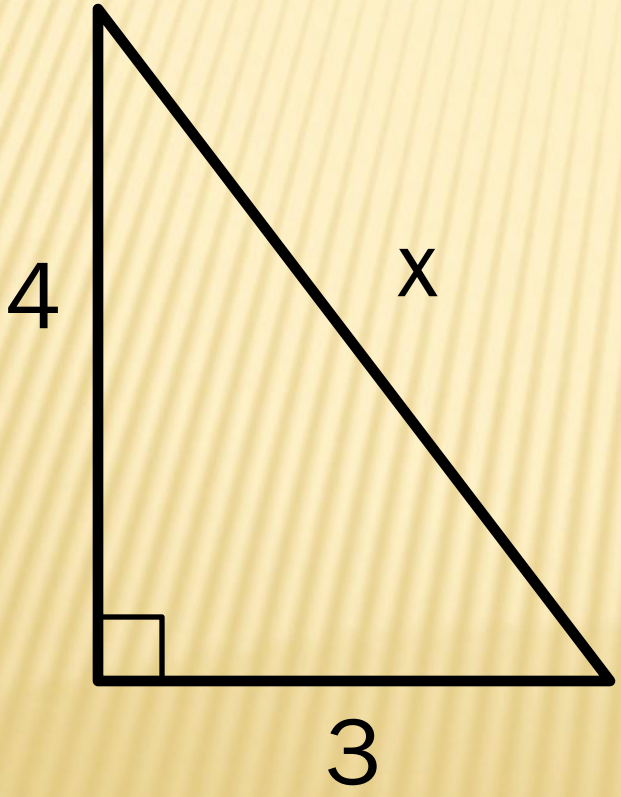


APPLICATION

Two examples of how to use the Pythagorean Theorem

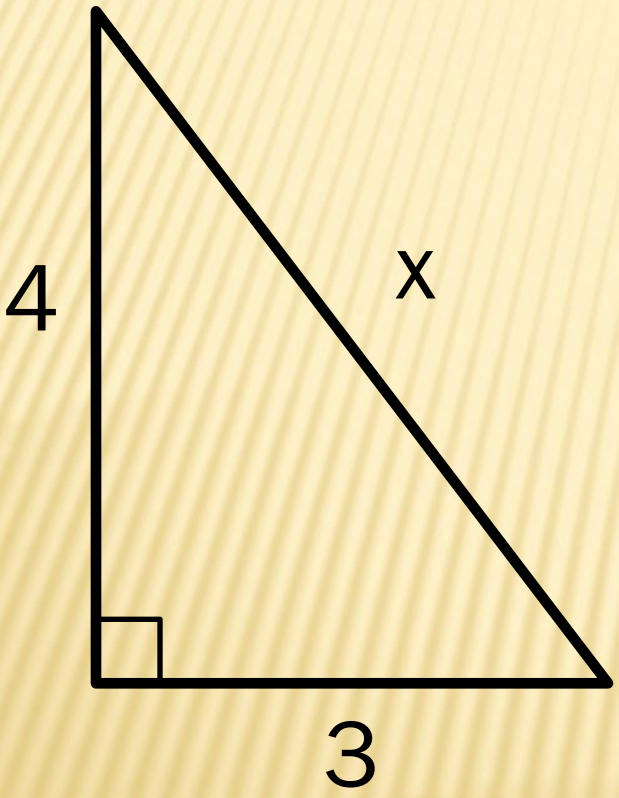
EXAMPLE: FINDING THE HYPOTENUSE

- Find the length of the hypotenuse of a right triangle with the given sides.



- Step 1: Identify the which side is the hypotenuse
- Step 2: Set up the formula
- Step 3: Solve for the unknown
- Step 4: Conclusion

EXAMPLE: FINDING THE HYPOTENUSE (CONT'D)



- Step 1:

"x" is the hypotenuse

- Step 2:

$$a^2 + b^2 = c^2 \rightarrow 4^2 + 3^2 = x^2$$

- Step 3:

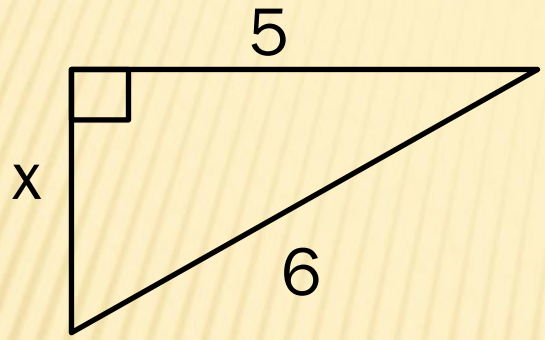
$$4^2 + 3^2 = x^2$$
$$16 + 9 = x^2$$
$$25 = x^2$$
$$\sqrt{25} = \sqrt{x^2}$$
$$5 = x$$

- Step 4:

The length of the hypotenuse is 5 units.

EXAMPLE: FINDING A LEG

- Find the length of the unknown side of the right triangle.



- Step 1:

Side with length "6" is the hypotenuse

- Step 2:

$a^2 + b^2 = c^2 \rightarrow x^2 + 5^2 = 6^2$

- Step 3:

$$x^2 + 5^2 = 6^2$$
$$x^2 + 25 = 36$$
$$-25 \quad -25$$
$$x^2 = 11$$
$$\sqrt{x^2} = \sqrt{11}$$
$$x \approx 3.32$$

- Step 4:

The length of the unknown side is ~3.32 units

COMMON MISTAKES

- Applying the Pythagorean Theorem when it is not a right triangle
- Forgetting “c” represents the hypotenuse
- Forgetting to take the square root when solving for the unknown

THE DISTANCE FORMULA

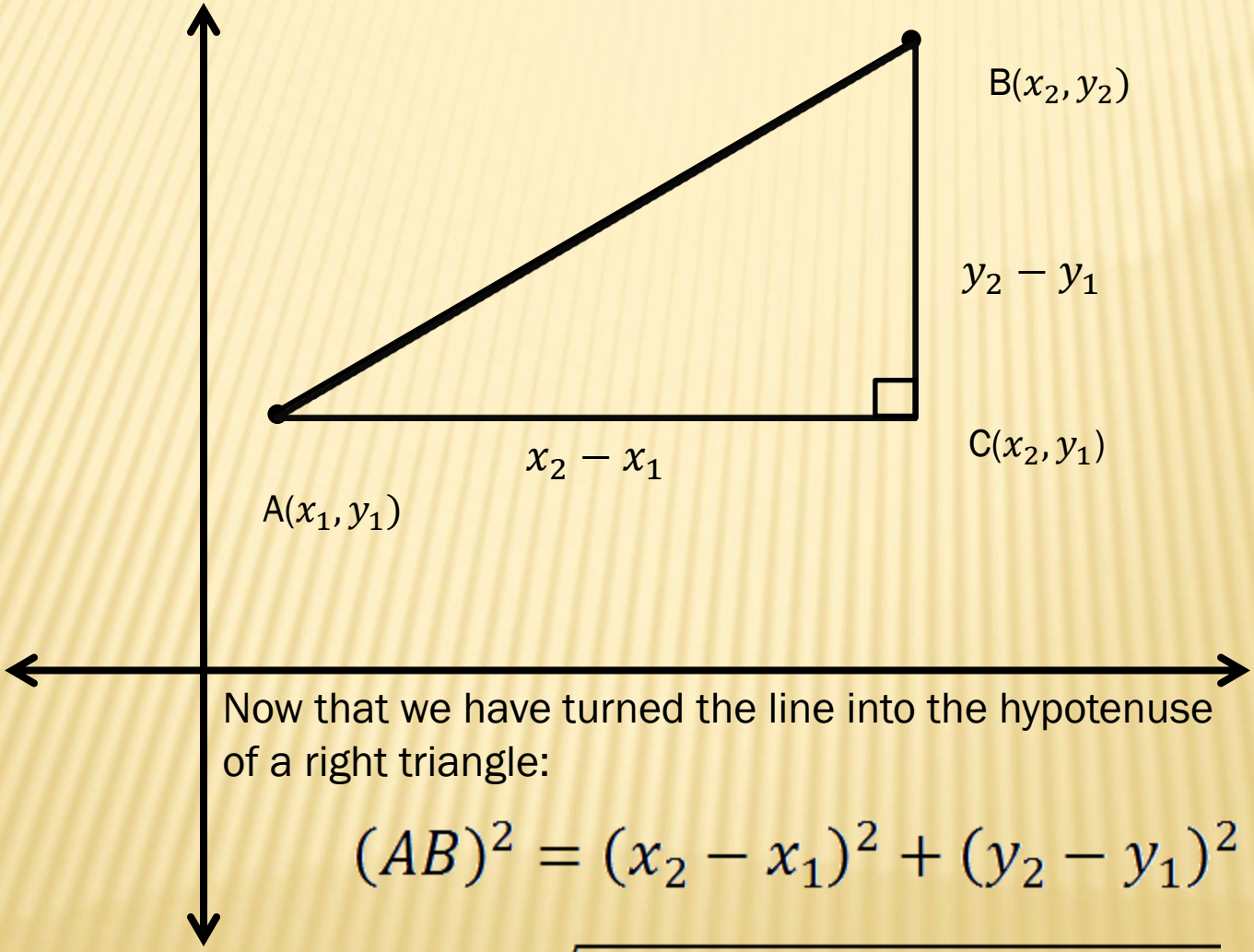
How to find the difference between two points in a coordinate plane

THE DISTANCE FORMULA

- If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

GEOMETRIC REPRESENTATION



Now that we have turned the line into the hypotenuse of a right triangle:

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

APPLICATION

One example of how to use the Distance Formula

EXAMPLE

- Find the distance from points F(1,0) and G(3,3).
- Step 1: Plot the points and draw a line to connect them.
- Step 2: Add legs to make a right triangle.
- Step 3: Use the Distance formula

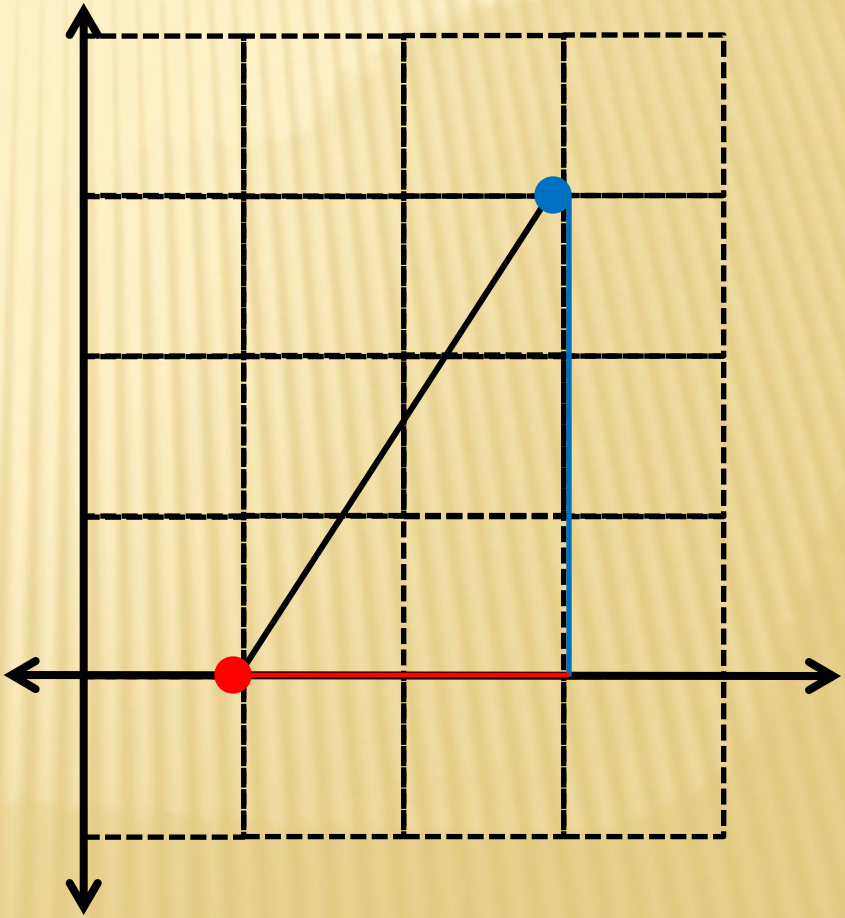
$$FG = \sqrt{(3-1)^2 + (3-0)^2}$$

$$FG = \sqrt{2^2 + 3^2}$$

$$FG = \sqrt{4+9}$$

$$FG = \sqrt{13}$$

$$FG \approx 3.61\ units$$



RETURN TO PROBLEM STATEMENT

Partner work and discussion

PARTNER WORK

- Find a partner, and find how far the boat in the problem statement drifted.
- When you think you have an answer, show your work on the board.
- Be prepared to answer one of the following questions:
  - How did you find the correct answer?
  - Where did you run into difficulties?

ASSIGNMENT: pp. 195-197 #'s 2-36, evens only

BEFORE YOU LEAVE:

Try to correctly write the Pythagorean Theorem and Distance Formula without looking

SOURCES

"Demonstrate the Pythagorean Theorem." *NOVA Online*. PBS, Nov. 2000. Web. 07 Mar. 2012

Larson, Ron, Laurie Boswell, and Lee Stiff. "The Pythagorean Theorem and the Distance Formula." *Geometry: Concepts and Skills*. Evanston, IL: McDougal Littell, 2003. 191-94. Print.