Algebra I:
Chapter 1 Unit Plan

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Education 352
Professor Schilling
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A. TEXTBOOK/COURSE INFORMATION

NAME OF COURSE: Algebra I

DESCRIPTION OF COURSE: This course has been created for a wide variety of students—mainly ranging from grades 8-10. It lays the foundation for all upper level mathematical courses. Its curriculum places much emphasis on creating and solving expressions that correlate to real-life mathematical problems.

NAME OF CHAPTER: Chapter 1

DESCRIPTION OF CHAPTER: This chapter, entitled “The Language of Algebra”, introduces students to the exciting world of algebra. To go along with teaching the basic language of algebra, much of this chapter is also focused on writing, understanding and evaluating numerical and algebraic expressions. Past mathematical concepts and properties will be reviewed and by the end of the chapter, students will have also learned a new problem solving technique and how to collect and display data.

TITLE OF TEXTBOOK: Algebra: Concepts and Applications

NAMES OF AUTHORS: Jerry Cummins, Carol Malloy, Kay McClain, Yvonne Mojica and Jack Price

NAME OF PUBLISHING COMPANY: Glencoe/McGraw-Hill

COPYRIGHT DATE: 2004

READING LEVEL OF TEXTBOOK: Eighth grade reading level
B. PHILOSOPHY OF READING IN THE CONTENT

STANDARDS:
A1.1.1 Compare real number expressions.

A1.1.3 Understand and use the distributive, associative, and commutative properties.

A1.9.1 Use a variety of problem solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards

A1.9.3 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations

A1.9.6 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

A1.9.7 Identify the hypothesis and conclusion in a logical deduction.

A1.9.8 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

IMPORTANCE: This unit is important to eighth, ninth and tenth grade students because having an understanding of the algebraic language can help students solve real-life problems. At this stage in their lives, these students are becoming young adults, and they need to know how to make important decisions on their own. Having a good understanding of algebra will help these decisions become easier. Some students will earn jobs that require them to understand the algebraic terminology in order to survive. Others will need to know how to create and solve algebraic expressions, so they can make good decisions when purchasing items on their own. After the unit, the students will have learned a new problem solving technique and be able to formulate different types of data relating to real-life situations.
PHILOSOPHY: As for any content area, reading is a necessary must as it not only gives students a better understanding of what is being taught, but also enlightens their minds and develops them into critical thinkers. In mathematics, the importance of reading tends to be overlooked. Many students feel that they can become successful mathematicians without being able to read, which is a very false conception. First and foremost, reading allows students to understand what they are learning, and then, they can apply this knowledge to solving mathematical problems. Reading is the core of any subject as all content revolves around a student’s ability to read. Without it, everything would look foreign, or simply blank.

Prior to becoming a collegiate student, I used to believe in the misconception mentioned above. I had simply thought that math only consisted of using numbers to solve problems. This may seem true; however, my college professors have taught me that math is not just using and analyzing numbers. They have taught me how to read math, which includes understanding terms and concepts, looking over examples and evaluating how mathematics can be applied to everyday life. Now, when I look back on every math course I have taken, I realize that reading has always been a part of them. To go along with reading definitions and concepts, mathematics also requires students to be able to read other elements such as instructions and word problems. Other than these elements involving words, math is represented by numbers and symbols. Many people feel that looking at numbers and symbols is not reading, but this is just another misconception. Success in mathematics, as for any subject, depends highly on a student’s ability to read well, and as a teacher, I will make sure my students know this.
C. READABILITY TEST

SAMPLE ONE:
There are many ways to represent numbers. One way to represent numbers is with a number line. The number line also shows the order of numbers; 2 is to the left of 3, so 2 is smaller than three. A negative number is a number less than zero. To include negative numbers on a number line, extend the line to the left of zero and mark off equal distances. Negative whole numbers are members of the set of integers. So, integers can also be represented on a number line. Sets of numbers can also be represented by Venn diagrams. The

Sentence Length: 8.1 sentences
Number of Syllables: 148 syllables

SAMPLE TWO:
The Caribbean islands have many different species of birds. To determine if there is a relationship between area and number of bird species, we can graph the data points in a scatter plot. In a scatter plot, two sets of data are plotted as ordered pairs in the coordinate plane. For example, the point with the box around it is at (95, 236). You can use the scatter plot to draw conclusions and make predictions about the date. The country with the least area has 350 species of birds. In general, as area increases, the number of species of

Sentence Length: 6.7 sentences
Number of Syllables: 151 syllables

SAMPLE THREE:
You have learned when and how to solve systems of equations by graphing, substitution, and elimination using addition or subtraction. The best times to use these methods are summarized in the table below. Sometimes neither of the variables in a system of equations can be eliminated by simply adding or subtracting the equations. In this case, another method is to multiply one or both of the equations by some number so that adding or subtracting eliminates one of the variables. Use elimination to solve the system of equations. Multiply the first equation by -4 so that the x terms are

Sentence Length: 5.8 sentences
Number of Syllables: 162 syllables

From these word-samples, the Fry Readability Test shows that this textbook has a 9th grade reading level. This textbook is used by mainly 8th, 9th and 10th grade students, so I am not surprised by the results of the test. However, I would not promote this test as the accurate way to measure the reading level of a book. It is based only on sentence length and the amount of syllables a given passage contains. These two categories tend to fluctuate throughout any book, and therefore, are not the most reliable categories to determine a book’s reading level. The test also neglects how difficult the content is of a book, and content definitely needs to be considered when determining reading level.
D. ANNOTATED LIST OF TRADE BOOKS FOR MATHEMATICS


*Once upon a Dime* takes the reader on a mathematical adventure to learn the concepts of money, estimation and measurement. It tells the tale of a farmer who discovers that a special tree on his farm produces different kinds of money. The type of money being produced depends simply on what fertilizer the farmer uses, and throughout the story, the farmer continuously counts how much money he has earned.


*Spaghetti and Meatballs for All* introduces some basic language and concepts of geometry. The story describes a couple who is planning a family reunion, and in order to successfully place all their guests, the couple must use basic geometric skills. To go along with applying the concepts of area, perimeter and forming shapes, the story also adds a pinch of humor to make this a pleasant read for any child or adolescent.


*What’s Your Angle, Pythagoras* applies the Pythagorean Theorem to solve problems involving right triangles. It is a fictional tale of the Greek philosopher and mathematician, Pythagoras. As a boy, he always becomes confused when trying to solve problems for his family. However, after a trip to Egypt and an encounter with a builder, Pythagoras thinks about right angles from a different perspective and devises the theory that contains his name today.


*Amanda Bean’s Amazing Dream* describes how multiplying numbers is a much more efficient process than simply counting numbers. In the story, Amanda Bean has a knack for counting anything and everything. Her teacher says that using multiplication would make her counting process much faster. Amanda does not become convinced with what her teacher says until she has a dream that overwhelms her with counting objects. This story will enlighten young minds to improve their multiplication skills.


*Sir Cumference and the First Round Table* describes a mathematical adventure set in the days of knights and chivalry. The well-known King Arthur and his knights discover a problem—their table is not of right shape. Sir Cumference, a knight of King Arthur’s, and his family help the king find the ideal shape for the table. Throughout the adventure, a variety of math skills are applied such as area, perimeter and circumference.
Trade books can enhance my classroom’s content because they apply mathematical concepts and vocabulary to real situations. Even though some of the books may be fictional, the concepts they teach are very real. In all subjects, especially mathematics, students are always wondering how the lessons they learn will actually be used in life. Trade books or stories show students how these lessons and concepts can be applied to a given situation. They also allow students to learn from outside their classrooms and textbooks.
E. LESSON PLAN TO ACTIVATE PRIOR KNOWLEDGE

MANCHESTER COLLEGE
Department of Education

LESSON PLAN by: Kyler Kearby

Lesson: Writing Expressions and Equations

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
Standard A1.1.1: Compare real number expressions.

Performance Objective:
When the students are given fifteen problems that involve writing algebraic expressions and equations, they will correctly write twelve out of the fifteen problems.

Assessment:
The students will be assigned fifteen homework problems. Depending on the problem, the students must either write an expression or an equation. The teacher will evaluate the students on whether or not the expressions and equations are written correctly.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it applies to entire classroom
- Clean chalkboard
- Fresh chalk
- Hard hat (Construction)
- Safety vest
- Chef’s hat
- Apron
- Lab Coat
- Safety Goggles
- Cowboy Hat
- Flannel shirt
- Piece of straw

Procedure:

Introduction/Motivation:
First, ask the students, “When your time as a student comes to an end, will you still need to use math and if so, can you give an example to support this claim?” (Bloom: Application). After each student has had a chance to respond, explain to them that math will still be used in the workplace, at home, at the store, etc. Introduce the new chapter, entitled, “The Language of Algebra” and describe how having a good understanding of
algebra will be beneficial in life after graduation. Say to the students, “In order to have a good understanding of algebra, a student must first understand its basic elements.” Then, tell the students to give definitions of an expression and also an equation. If they give accurate definitions, continue with the lesson. Otherwise, explain to the students that an expression only contains numbers and mathematical operations, while an equation is a mathematical sentence that contains an equals sign. Be sure to point out that the main difference between the two is that an equation contains an equals sign, where as an expression does not contain one.

**Step-by-Step Plan:**

1. Say to the students, “In life, you definitely need to know how to write expressions and equations. Even though they may not always be written down on paper, they will be a part of the thinking process when you are solving a problem.” Put on the hard hat and safety vest. Tell the students that you are a construction laborer and your foreman wants you to figure out how much square feet of concrete will be needed to fill in two areas. Point out to them that the foreman is just asking you to find the sum of the two areas and that one area is definitely 20 square feet. Write “the sum of 20 and n” on the board and explain to the students that this represents the given problem. Tell them that you want to rewrite this as an algebraic expression. Ask the students, “What mathematical operation is used to find the sum of two values?” (Bloom: Knowledge). If they say “addition”, tell them they are correct and go on with the lesson. If they give a different response, explain to them why addition would be used. Write “20 + n” on the board and explain to the students that they have now written an algebraic expression. (Gardner: Logical/Mathematical, Visual-Spatial and Interpersonal)

2. Take off the construction outfit and put on the chef’s hat and apron. Tell the students that your boss wants you to bake seven less cakes than the amount you baked yesterday. On the board, write this in mathematical terms—“the difference of k and 7”. Tell them that you want to rewrite this as an algebraic expression. Ask the students, “Does the difference of two values mean to subtract or add the two values?” (Bloom: Comprehension). If they say “subtract”, then continue on with the problem. If they give a different response, explain to them why it should be “subtract”. Explain to the students that you would subtract 7 from k, which produces the algebraic expression “k – 7”. (Gardner: Logical/Mathematical, Visual-Spatial and Interpersonal)

3. Take off the cooking gear and pull out the lab coat and safety goggles. Ask for a student to volunteer and be the scientist. Inform the students that the scientist is filling a beaker with two chemicals, and the chemicals cannot be over spilled. Tell them that 5 mL of the 1st chemical has already been poured into the 20 mL beaker and the goal is to determine exactly how much of the 2nd chemical needs to be poured in. On the board, write “five plus x equals twenty” to represent the given situation. In order to write this as an equation, tell the students it must be translated from word form into numbers and symbols. Therefore, the equation would be “5 + x = 20”. (Gardner: Logical/Mathematical, Visual-Spatial and Bodily-Kinesthetic)

4. Have the student take off the lab coat and safety goggles and pull out the flannel shirt and cowboy hat. Again, ask for a student to volunteer and be the farmer. Also, give the student the piece of straw to put in his/her mouth. Explain to the students that the farmer wants to plant some corn on his/her 12 acre farm. Also, tell them that the
width of the field is 4 acres, but its length has yet to been measured. To represent this situation, write “the product of four and y equals twelve” on the board. Ask the students, “What does the product refer to in terms of a mathematical operation?” (Bloom: Knowledge). If they say multiplication, tell them they are correct. Otherwise, explain to them that “product” means to multiply. Write the equation on the board, which is “4y = 12”. (Gardner: Logical/Mathematical, Visual-Spatial, Interpersonal and Bodily-Kinesthetic)

5. Have the student return the flannel shirt and cowboy hat and throw away the piece of straw. Ask the students if they have any questions about writing expressions and equations. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (14 – 22 all and 29 – 34 all) on p. 7. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

**Closure:**
With about five minutes left of class, say to the students, “You should now understand the definition of an expression fairly well. From class examples and the homework problems, you have worked with expressions that only include one mathematical operation.” Ask them, “Can an expression contain more than one mathematical operation, and if so, would the order of each operation affect the result?” (Bloom: Analysis)

**Adaptations/Enrichment:**

**Student with ADHD:**
Give the student breaks throughout the class period and make sure you spread them out. Request the student to take a break, but do **not** demand the student to take a break.

**Student with Traumatic Brain Injury (TBI):**
Give the student extra time to process the given information. Place the student’s desk away from distractions such as windows, a pencil sharpener, doorway, etc. Repeat instructions and avoid making the student focus for a long period of time.

**Self-Reflection:**
Do the students understand how to write expressions and equations?  
Do the students know the difference between an expression and an equation?  
Did I keep the students engaged throughout the entire lesson?  
Are the students participating in classroom discussions?
F. LESSON PLAN TO INTRODUCE NEW VOCABULARY

MANCHESTER COLLEGE
Department of Education

LESSON PLAN by: Kyler Kearby

Lesson: Understanding order of operations

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
A1.9.3: Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving operations.

Performance Objective: When the students are given twenty-five problems that involve identifying mathematical properties and using the order of operations to evaluate algebraic expressions, they will correctly identify or solve twenty out of the twenty-five problems.

Assessment:
The students will be assigned twenty-five homework problems and a quiz—which will be issued at a later date. Depending on the problem, the students must either identify a mathematical property or evaluate algebraic expressions using the order of operations. When identifying a mathematical property, the teacher will evaluate the students on whether or not the correct property has been written. When evaluating algebraic expressions, the teacher will evaluate the students on not only whether or not the answers are correct, but also on the work shown. For the quiz, the teacher will evaluate the students on whether or not the given properties have been assigned to the correct problems.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it applies to entire classroom
- Type and print off enough student copies of the “Chapter One Properties” handout (attached)
- Type and print off a teacher copy of the “Chapter One Properties” handout (attached)
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “In our previous lesson, we learned how to correctly write algebraic expressions and equations. Today, the goal is to learn how to evaluate these expressions, but first, we must learn a few properties that will help us gain a better understanding of mathematics, in general.” Ask the students, “What exactly is a mathematical property? Define it for me” (Bloom: Knowledge). If the students do not give a correct definition, then explain to them that a property is a statement that is true for
any number. Tell the students that understanding mathematical properties is similar to having a good vocabulary in math or rather, for any subject. Next, pass out the “Chapter One Properties” handout to the students. (Also, get out the teacher’s copy of this handout as it may be needed for writing the properties and their contents on the board.) Say to the students, “Throughout this chapter, I will periodically define and exemplify mathematical properties, and as students, you will be expected to write down these definitions and examples on your handouts. Today, we will learn the first seven properties listed on the handout, and in the near future, you will be quizzed over these seven properties.” Looking at the teacher’s handout, write down the name and symbolic definition of the first property on the board. Give an explanation of the property and ask the students, “Using numbers, could anyone give me an example of this property?” (Bloom: Comprehension). If a student gives a correct example, then write it on the board. Otherwise, give the example listed on the handout and explain to the students why this exemplifies the property. For each property, repeat this process of defining the property and asking the students to give an example of it. After going over the properties, say to the students,” At the end of our previous lesson, I asked you a couple of questions. I asked can an expression contain more than one mathematical operation, and if so, would the order of each operation affect the result? Well, the answer to both questions is yes.”  Tell the students that today’s lesson is going to focus on solving expressions that contain more than one operation. The students should already be very familiar with this concept, so explain to the students that the lesson should be review for them.

**Step-by-Step Plan:**

1. Write “Please Excuse My Dear Aunt Sally” on the board and be sure to capitalize and underline the first letter of each word. Explain to the students that this acronym, or catchy phrase, is used to remind us how to solve an expression that contains more than one operation. Erase everything, except for the first letter of each word, and ask the students, “In terms of mathematical operations, what does each of these letters stand for?” (Bloom: Knowledge). They students should say that “P” represents parenthesis, “E” represents exponential, “M” represents multiplication, “D” represents division, “A” represents addition and “S” represents subtraction. If they do not give an answer such as this, then explain to them what each letter represents. Explain, or rather remind the students that the order these letters are in correlates to which operations should be done first when solving these types of expressions. Using the letters that are on the board, have the students come up with their own acronyms and tell them to write them on a sheet of paper. To go along with this, have them draw a picture of their acronym. (Gardner: Logical/Mathematical, Interpersonal, Verbal-Linguistic and Visual-Spatial)

2. Let some of the students share their acronyms with the entire classroom. Then, write the expression $38 – 5 \times 6$ on the board. Ask the students, “In order to solve this expression, should the first step be to multiply 5 and 6 or subtract 5 from 38?” (Bloom: Comprehension). If the students say to multiply, then go on with the problem; otherwise, tell them to look at their acronyms and they will notice that multiplication comes before subtraction. After multiplying 5 and 6, the expression now reads $38 – 30$. The last step is to simply subtract 30 from 38 and the answer is 8. (Gardner: Logical/Mathematical and Interpersonal)
3. Write $5(3 + 6) ÷ 9$ on the board. Ask the students, “According to our acronyms, we should first solve what is inside the parenthesis, right?” (Bloom: Comprehension). If the students say yes, then continue solving the expression; otherwise, explain to them why they are wrong. After solving what is inside the parenthesis, the expression now reads $5 \times 9 ÷ 9$. Explain to the students that when one operation is multiplication and the other is division, then order does not matter—the same goes for an addition operation with a subtraction operation. When this occurs, remind the students that it is best to first evaluate the easier part of the expression. Ask the students, “Would it be easier to solve for the product of 5 and 9 or for the quotient of 9 and 9?” (Bloom: Analysis). If the students stay to find the quotient of 9 and 9, then continue solving the problem; otherwise, explain to them the dividing 9 by 9 is much easier then multiplying 5 by 9. After solving $9 ÷ 9$, the expression now shows $5 \times 1$. Evaluate this final expression and the answer is 5. (Gardner: Logical/Mathematical and Interpersonal)

4. Write $20 – [5(2 + 1)]$ on the board. Explain to the students that the brackets shown in this expression represent the same operation as set of parenthesis. Tell them that when an expression has two sets of parenthesis, then the outside set is usually changed to a set of brackets. Explain to them that when this occurs, the expression inside the parenthesis is evaluated first and then the new expression inside the set of brackets is evaluated next. With this in mind, evaluate $2 + 1$ and the expression now becomes $20 – [5 \times 3]$. As stated, the next step is to evaluate what is inside the set of brackets, which leaves the expression of $20 – 15$. Solve for this expression and the answer is 5. (Gardner: Logical/Mathematical).

5. Ask the students if they have any questions about identifying these new properties or solving expressions using the order of operations system. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (19 – 33 all and 40 – 49 all) on pgs. 11-12. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

**Closure:**

With a few minutes left in the class period, say to the students, “You should now feel very comfortable with solving expressions that contain multiple operations. Today, you learned seven properties and we will continue to learn more throughout this chapter.” Ask the students, “Prior to this course, can you recall any other properties that you have learned?” (Bloom: Knowledge)

**Adaptations/Enrichment:**

**Student with English as Second Language:**
Have a translator available for the student. Be sure to frequently verify with the translator that the student comprehends the material.

**Student with Gifts and Talents in Creativity:**
Have the student create a ten problem worksheet that underlines the concept of evaluating expressions using the order of operations concept. This worksheet could then be used as a review assignment at the end of the chapter.
**Self-Reflection:**
Do the students understand how to evaluate expressions using the order of operations concept?
Do the students understand the different properties that were taught?
Did I keep the students engaged throughout the entire lesson?
Were the students willing to participate in classroom activities or discussions?
## Chapter One Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbolic Definition</th>
<th>Numerical Example</th>
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</thead>
<tbody>
<tr>
<td>Substitution</td>
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<tr>
<td>Reflexive</td>
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<td>Symmetric</td>
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<tr>
<td>Transitive</td>
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<tr>
<td>Additive Identity</td>
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<tr>
<td>Multiplicative Identity</td>
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<tr>
<td>Multiplicative Property of Zero</td>
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<td>Commutative Property of Addition</td>
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<td>Commutative Property of Multiplication</td>
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<td>Associative Property of Addition</td>
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<td>Associative Property of Multiplication</td>
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<tr>
<td>Distributive Property</td>
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<tr>
<td>Property</td>
<td>Symbolic Definition</td>
<td>Numerical Example</td>
</tr>
<tr>
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<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>Substitution</td>
<td>If ( a = b ), then ( a ) may be replaced by ( b ).</td>
<td>If ( 9 + 2 = 11 ), then ( 9 + 2 ) may be replaced by ( 11 ).</td>
</tr>
<tr>
<td>Reflexive</td>
<td>( a = a )</td>
<td>( 10 = 10 )</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If ( a = b ), then ( b = a ).</td>
<td>If ( 10 = 4 + 6 ), then ( 4 + 6 = 10 ).</td>
</tr>
<tr>
<td>Transitive</td>
<td>If ( a = b ) and ( b = c ), then ( a = c ).</td>
<td>If ( 3 + 5 = 8 ) and ( 8 = 2(4) ), then ( 3 + 5 = 2(4) ).</td>
</tr>
<tr>
<td>Additive Identity</td>
<td>For any number ( a ), ( a + 0 = 0 + a = a ).</td>
<td>( 45 + 0 = 45 )</td>
</tr>
<tr>
<td>Multiplicative Identity</td>
<td>For any number ( a ), ( a \times 1 = 1 \times a = 1 ).</td>
<td>( 12 \times 1 = 12 )</td>
</tr>
<tr>
<td>Multiplicative Property of Zero</td>
<td>For any number ( a ), ( a \times 0 = 0 \times a = 0 ).</td>
<td>( 7 \times 0 = 0 )</td>
</tr>
<tr>
<td>Commutative Property of Addition</td>
<td>For any numbers ( a ) and ( b ), ( a + b = b + a ).</td>
<td>( 5 + 7 = 7 + 5 )</td>
</tr>
<tr>
<td>Commutative Property of Multiplication</td>
<td>For any numbers ( a ) and ( b ), ( a \times b = b \times a ).</td>
<td>( 3 \times 10 = 10 \times 3 )</td>
</tr>
<tr>
<td>Associative Property of Addition</td>
<td>For any numbers ( a ), ( b ) and ( c ), ( (a + b) + c = a + (b + c) ).</td>
<td>( (24 + 8) + 2 = 24 + (8 + 2) )</td>
</tr>
<tr>
<td>Associative Property of Multiplication</td>
<td>For any numbers ( a ), ( b ) and ( c ), ( (a \times b) \times c = a \times (b \times c) ).</td>
<td>( (9 \times 4) \times 8 = 9 \times (4 \times 8) )</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>For any numbers ( a ), ( b ) and ( c ), ( a(b + c) = ab + ac ) and ( a(b - c) = ab - ac ).</td>
<td>( 2(5 + 3) = (2 \times 5) + (2 \times 3) ) ( 2(5 - 3) = (2 \times 5) - (2 \times 3) )</td>
</tr>
</tbody>
</table>
Properties Quiz

Instructions: Match each property with the problem that demonstrates it. (Each question is worth one point each.)

1. __: \(21 = 17 + 4\), therefore \(4 + 17 = 21\).
2. __: \(7 = 7\)
3. __: \(0 + 150 = 150\)
4. __: \(65 \times 0 = 0\)
5. __: \(15 - 6 = 9\), therefore \(15 - 6\) may be replaced by 9.
6. __: \(2000 \times 1 = 2000\)
7. __: \(7 + 5 = 12\) and \(3(4) = 12\), therefore \(7 + 5 = 3(4)\).

a. Transitive
b. Substitution
c. Multiplicative Property of Zero
d. Reflexive
e. Multiplicative Identity
f. Additive Identity
g. Symmetric
LESSON PLAN by: Kyler Kearby

Lesson: Understanding commutative and associative properties

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
A1.1.3: Understand and use the distributive, associative, and commutative properties.

Performance Objective: When the students are given twelve problems that involve identifying the commutative and associative properties, they will correctly identify ten out of the twelve problems.

Assessment:
The students will be assigned twelve homework problems. For each problem, the students must identify the given property as either associative or commutative and if asked, they must also simplify a given expression. The teacher will evaluate the students on whether or not the properties have been correctly identified and also whether or not a given expression has been simplified.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for a student with ADHD
- Have the teacher copy of the “Chapter One Properties” handout
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “As I mentioned in our previous lesson, we will be learning some basic and essential mathematical properties over the next few days. Today, we will learn about the associative and commutative properties. In order to introduce these properties, I am going to ready a children’s book that I am sure many of you read as a kid.” Read the book, I Know an Old Woman Who Swallowed a Fly. After reading the book, ask the students, “How might this book relate to the properties that we are going to discuss today?” (Bloom: Analysis). Tell the students to think about the order in which everything was eaten and ask them, “I mean, could the spider have been swallowed after the bird to catch the bird?” (Bloom: Comprehension). Make sure the students know that this could not have happened, which means that order does matter. Say to the students,
“However, if the properties of commutative and associative were used in this story, then the spider could have been swallowed after the bird, the cow could have been swallowed after the horse and so forth.” Show this example symbolically on the board. Ask the students, “With this in mind, does order matter when using either of these properties?” (Bloom: Comprehension). Explain to them that it does not matter as the order never changes the final outcome.

Step-by-Step Plan:

1. Have the students get out their properties handout. Looking at the teacher’s handout, write down the name and symbolic definition of the Commutative Property of Addition. Give an explanation of the property and ask the students, “Using numbers, could anyone give me an example of this property?” (Bloom: Comprehension). If a student gives a correct example, write it on the board. Otherwise, give the example listed on the handout and explain to the students why this exemplifies the property. For each of the associative and commutative properties listed, repeat this process of defining the property and asking the students to exemplify it. (Gardner: Logical/Mathematical, Interpersonal, and Verbal-Linguistic)

2. Make sure the students know that the only difference between the two properties is that two numbers are used in the commutative properties, while three numbers are used in the associative properties. On the board, write $8 + 9 = 9 + 8$. Ask the students, “Which property is being displayed by this statement?” (Bloom: Knowledge). Make sure they understand that the Commutative Property of Addition is displayed and replace the addition signs with multiplication signs. Tell the students this statement now represents the Commutative Property of Multiplication. (Gardner: Logical/Mathematical and Interpersonal)

3. Write $7 \times (8 \times 6) = (7 \times 8) \times 6$ on the board. Ask the students, “Does this statement represent a commutative or an associative property?” (Bloom: Comprehension). Explain to the students that it represents an associative property—the Associative Property of Multiplication—as it groups three numbers together. (Gardner: Logical/Mathematical and Interpersonal)

4. Write $(k \times 3) \times 5$ on the board. Tell the students that they must simplify this expression and also state the property that will be while simplifying it. Explain to the students that, by the Associative Property of Multiplication, this expression can be written as $k \times (3 \times 5)$. Multiply and the simplified expression is $15k$. (Gardner: Logical/Mathematical)

5. Ask the students if they have any questions about identifying the commutative and associative properties or using them to simplify expressions. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (10 – 21 all) on pg 17. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)
Closure:
With a few minutes left in the class period, say to the students, “I hope you are getting very familiar with these properties that we have learned. We will continue to learn more, and as I mentioned the other day, there will be a quiz over these properties.” Explain to the students that it would be wise for them to study the properties for five minutes each night. Remind the students that the commutative and associative properties must use the operations of addition and multiplication separately as they cannot contain a combination of the two in a given statement. Ask them, “Is there a mathematical property that does contain a combination of both operations?” (Bloom: Knowledge)

Adaptations/Enrichment:

Student with ADHD:
1) Give the student breaks throughout the class period and make sure you spread them out. Request the student to take a break, but do not demand the student to take a break.
2) Ask the student a question or two throughout the lesson. Make sure to warn the student in advance—use a technique such as tapping the student on the shoulder, walking in front of the student, etc.—to let the student comprehend the question better and prepare his or her answer.
3) Record your lesson on audiotape and give it to the student.
4) Make sure the student is displaying good organizational skills and check to see if he or she has written down today’s assignment.

Self-Reflection:
Do the students understand the properties that were taught?
If given a quiz right now, could the students correctly identify a similarity between the associative and commutative properties and also a difference?
Did I keep the students engaged throughout the entire lesson?
Did the students participate in classroom discussions?
Lesson Plan

Understanding the Distributive Property

Lesson Plan by: Kyler Kearby

Lesson: Understanding the distributive property

Length: 70 minutes  
Grade Intended: 9th and 10th

Academic Standard:
A1.1.3: Understand and use the distributive, associative, and commutative properties.

Performance Objective: When the students are given eighteen problems that involve applying the distributive property to simplify expressions, they will correctly simplify fifteen out of the eighteen problems.

Assessment:
The students will be assigned eighteen homework problems. For each problem, the student must use the distributive property to simplify the given expression. The teacher will not only evaluate the students on whether or not the expressions have been correctly simplified, but also whether or not the distributive property has been correctly used.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for students with learning disabilities
- Have the teacher copy of the “Chapter One Properties” handout
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “Over the past two lessons, we have learned several fundamental properties of mathematics, and we will learn another one today. I have stated there will be a quiz over the first seven properties listed on your “Chapter One Properties” handout, and I decided to issue this quiz at the beginning of our next class session. At the end of today’s lesson, we will review the properties that will be on the quiz.” Ask the students if they have any questions in regards to the quiz. Then, say to the students, “At the end of our previous session, I asked you to think about if there is a mathematical property that contains a combination of both the addition and multiplication operations. Well, there is a property that meets this criteria, and it is the distributive property.” Ask the students, “What is the definition of “distribute”? Confirm your definition by using the term in a sentence” (Bloom: Application). Make sure the students know that “distribute” basically means to “pass out” or “deliver” and an example of a sentence using the term could be “The teacher will distribute the quiz tomorrow.” Explain
to the students that knowing the definition of “distribute” will be very helpful in understanding the distribution property.

**Step-by-Step Plan:**

1. Have the students get out their properties handout. Looking at the teacher’s handout, write down the name and symbolic definition of the Distributive Property. Explain to the students that the distributive property simply implies that an integer or variable can be distributed to a quantity of integers or variables. Say to the students, “For example, I will be distributing twenty-five quizzes, but what happens if two students are absent? This question can be written as a mathematical expression in which the distributive can be used.” Write the expression 1(25 – 2) on the board. Make sure the students understand that “1” represents a quiz, “25” represents the number of students in the class and “2” represents the students who will be absent. Ask the students, “How would I solve this problem?” (Bloom: Application). Have the students guide you through the steps and they should come up with an answer of 23. They should first compute what is inside the parenthesis and then multiply that answer by 1, so 25 – 2 = 23 x 1 = 23. (Gardner: Logical/Mathematical, Interpersonal and Verbal-Linguistic)

2. Say to the students, “This is the correct answer, but there are actually two ways to solve the problem. You solved it by simply using the order of operations, but the other way to solve it involves applying the distributive property. To make this easier to see, I will use a variable “y” to represent how many students will be absent.” Write the expression 1(25 – y) on the board. Ask the students, “Can you subtract an unknown variable from an integer?” (Bloom: Knowledge). Make sure the students know that this cannot happen. Also, make sure the students know that they cannot add or divide integers and variables with each other, but they can multiply them. Explain to the students that this expression can be simplified, though, by using the distributive property. Tell the students that “1” can be distributed to “25” and “y” by multiplication. Write 1(25) – 1(y) on the board and make sure the students understand this step. Explain to the students to now follow the order of operations, and therefore, 1 x 25 = 25 and 1 x y = y. The expression is simplified as 25 – x, and thus, 25 – x quizzes will be distributed. (Gardner: Logical/Mathematical and Interpersonal)

3. Ask the students, “Now, by using the distributive property, how would I solve the first problem of 1(25 – 2)?” (Bloom: Application). Have the students lead you through the problem. Make sure they first distribute “1” to the quantity of “25 – 2”, so the expression would then read 1(25) – 1(2). The students should then use the order of operations and again come up with the answer of 23 as 1 x 25 = 25, 1 x 2 = 2 and then 25 – 2 = 23. Ask the students, “You have now solved this problem using two different methods. Which method would you rather use and why?” (Bloom: Evaluation). If the students say that the first method seems quicker to do, agree with them, but say that it only saves them one step in comparison to the method using the distributive property. Remind the students, though, that the distributive property must be used when the expression contains a variable. (Gardner: Logical/Mathematical and Interpersonal)

4. Write 3(2n + 7) on the board. Ask the students, “What is the first step in simplifying this expression?” (Bloom: Knowledge). Once the students say to distribute “3” to the quantity of “2n + 7”, perform this operation. The expression should now read 3(2n) + 3(7). Have the students explain what to do next. They should say to multiply 3 by 2n and also 3 by 7, which produces the expression of 6n + 21. Ask the students, “Can you add an integer and a variable?” (Bloom: Knowledge). Verify to the students that it cannot happen, and thus, the expression is simplified. (Gardner: Logical/Mathematical and Interpersonal)
5. If the students put away their “Chapter One Properties” handout, have them get it back out. On the board, write an example of one of the first seven properties listed on the handout. Ask the students which property it exemplifies and repeat the process for each of the properties that will be on the quiz. Tell the students that the quiz will consist only of matching and remind them that it will be issued at the beginning of the next class. (Gardner: Logical/Mathematical and Interpersonal)

6. Ask the students if they have any questions in regards to either the quiz or simplifying expressions using the distributive property. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (15 – 32 all) on pg 22. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

**Closure:**
With a few minutes left in the class period, say to the students, “I hope you are beginning to feel confident in understanding and applying these properties. They are not only essential parts of this course, but also for any higher-level mathematics course.” Once again, remind the students to study for their quiz to go along with finishing their assignment at home. Ask them, “Speaking of home, how do you tend to solve problems when you are at home or rather, when you are away from school? These problems can relate to anything. For example, how do you solve the problem of a sibling arguing with you?” (Bloom: Knowledge). Explain to the students that they do not have to think mathematically about this.

**Adaptations/Enrichment:**

**Student with a Learning Disability:**

1) Give clear and precise instructions. Never assume a child understands something as long as they can repeat what you said. For this lesson, all they need to do is simplify expressions using the distributive property.

2) Vary pitch and tone of voice throughout the lesson. When asked a question, give the student five to six seconds to respond.

3) Do not grade handwriting and be more lenient with sloppy work. For the assignment, be lenient if the work is unorganized.

4) Have the student turn his or assignment sideways or let the student use colored pencil.

**Self-Reflection:**

Do the students understand how the term “distribute” relates to the distributive property?

Do the students understand how to use the distributive property to simplify expressions?

Did I keep the students engaged throughout the entire lesson?

Are the students sufficiently prepared for the next class session’s quiz?
I. LESSON PLAN MODIFIED FOR HIGH-ABILITY

MANCHESTER COLLEGE
Department of Education

LESSON PLAN by: Kyler Kearby

Lesson: Using a four-step plan to solve problems

Length: 70 minutes                         Grade Intended: 9\textsuperscript{th} and 10\textsuperscript{th}

Academic Standard:
A1.9.1: Use a variety of problem solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Performance Objective: When the students are given ten problems that involve applying the four-step plan to solve problems, they will correctly solve eight out of the ten problems.

Assessment:
The students will be assigned ten homework problems. For each problem, the student must use the four-step plan to solve each problem. The teacher will not only evaluate the students on whether or not the problems have been correctly solved, but also whether or not the four steps have been correctly used.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for high-ability students
- Have a copy of the “Properties Quiz” for each student (attached)
- Have a teacher’s copy of the “Properties Quiz” (attached)
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, as the students walk in the room, tell them to sit down quietly and get ready for the “Properties Quiz”. Once the students have sat down, pass out a quiz to each student. Read the instructions to them and make sure they clearly understand what they are supposed to do. Have the students turn their quizzes over once they have finished and collect them after the quizzes have all been turned over. The students should not take any longer than five minutes to finish the quiz. Next, say to the students, “The other day I told you think about how you solve problems that take place away from school. I sure most of you solve your problems differently than your peers, which is fine. There are many ways to solve problems, and today we will learn a problem solving strategy—the four step plan—that is not only used in mathematics, but also in real-life.” Identify and briefly explain a problem that you have solved in real-life such as choosing the college you attended. Tell the students that you had to use the four-step plan to solve this problem.
Step-by-Step Plan:

1. Identify and explain each of the four steps used in this strategy—explore, plan, solve and examine. Ask the students, “How would you explore the problem of choosing the college you want to attend?” (Bloom: Application). Listen to their answers and tell them that you explored this problem by visiting and researching different colleges and universities that you were interested in possibly attending. Tell the students that the next step was to make a plan—which was to list the pros and cons of each school. Explain to them that you used this list to choose which college had the most benefits and would be the best fit for you. Tell the students that the last step was to examine your choice and check to see if that was really the school for you. (Gardner: Logical/Mathematical and Interpersonal)

2. On the board, write “1, 2, 4, 8, 16, __, __. Find the next two numbers of this sequence.” Below this, write each of the four steps. Explain to the students that for their assignment they will be expected to list each step and explain what they did or how they performed the steps. Tell the students that the “explore” step has been given as the problem states what needs to be found. Ask the students, “What plan should we use to solve this problem? Would it be best to look for a pattern, draw a diagram, work backward, guess and check, or use some other strategy?” (Bloom: Comprehension). The students should say a pattern as sequences contain patterns. Ask the students, “In this sequence, what is the pattern?” (Bloom: Analysis). If the students are unable to identify the pattern, explain to them that it is to multiply the first number by two to get the next number and so on. With this mind, explain to the students that they simply need to multiply 16 by 2, which gives the next number. Then they need to multiply this new number by 2 to get the final number they need. So, $16 \times 2 = 32$ and $32 \times 2 = 64$, and therefore, the next two numbers of the sequence are 32 and 64. On the board, fill in the sequence with these numbers and ask the students whether or not they accurately complete the sequence. (Gardner: Logical/Mathematical and Interpersonal)

3. On the board, write “How many ways can you make 25 cents using dimes, nickels and pennies?” Tell the students that the goal is to simply find as many different ways possible to make 25 cents using any type of change, except quarters. Remind the students that the next step is to come up with a plan. Ask them, “For a plan, what problem solving strategy would work out very well in this situation?” (Bloom: Comprehension). Explain to them that creating a chart would be the ideal strategy if they had not already chosen it. Create two columns on the board labeled “Coin” and “Number”. Under the “Coin” heading, list dimes, nickels and pennies. Have the students come up to the board and list the different ways that 25 cents could be made from these types of coins. The students should come up with 12 different possible ways (The final table should resemble Table 1). Tell the students to check one final time if there are any other possible ways to make 25 cents from dimes, nickels and pennies. (Gardner: Logical/Mathematical, Interpersonal, Visual-Spatial and Bodily-Kinesthetic).
Table 1

<table>
<thead>
<tr>
<th>Coin</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimes</td>
<td>2 2 1 1 1 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Nickels</td>
<td>1 0 3 2 1 0 5 4 3 2 1 0</td>
</tr>
<tr>
<td>Pennies</td>
<td>0 5 0 5 10 15 0 5 10 15 20 25</td>
</tr>
</tbody>
</table>

4. Ask the students if they have any questions about using the four-step plan to solve problems. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (3 – 12 all) on pgs. 27-28. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

Closure:
With a few minutes left in the class period, say to the students, “I hope you leave today knowing that you could use this four-step plan to solve any problems that you may come across today or anytime in your lives.” Have them think about how this plan could have been used to solve past problems. Tell the students that problem solving is a form of logical reasoning. Ask them, “Speaking of logical reasoning, what are the two different types of logical reasoning?” (Bloom: Knowledge)

Adaptations/Enrichment:

**Students with high-abilities:**
Throughout this chapter, I will be sure to ask the high-ability students upper-level Bloom’s questions. High-ability students need to be challenged mentally and asking them questions that relate to knowledge and comprehension will simply not give them what they need. These students need to be analyzing, synthesizing and evaluating the different properties, problem-solving strategies and data of this chapter.

High-ability students have a much greater ability of developing their own ideas, instead of accepting others’ whether they are right or wrong. With this in mind, I need to push the high-ability students to develop their own ideas and beliefs. In this lesson, I could have them develop their own plan for solving problems. This would allow them to think outside the box and create a strategy that previously did not exist.

Another possible enrichment that relates to having these high-ability students develop their own ideas is having them create surveys of their choice. The second half of this chapter focuses on collecting and interpreting data; but rather than recording and analyzing other people’s data, they could analyze their own. These students would survey their peers on an issue, develop a graph or chart based on this information and finally make conclusions about what they had found. This idea would not only keep them engaged in the lessons, but it would more importantly allow them to gain hands-on experience in applying what they had learned.
Self-Reflection:
Do the students understand each step in the four-step plan of problem solving?
Do the students understand how to use the four-step plan to solve problems?
Did I keep the students engaged throughout the entire lesson?
Were the students willing to participate in classroom discussions?
Properties Quiz

Instructions: Match each property with the problem that demonstrates it. (Each question is worth one point each.)

8. ___: $21 = 17 + 4$, therefore $4 + 17 = 21$.

9. ___: $7 = 7$

10. ___: $0 + 150 = 150$

11. ___: $65 \times 0 = 0$

12. ___: $15 - 6 = 9$, therefore $15 - 6$ may be replaced by $9$.

13. ___: $2,000 \times 1 = 2,000$

14. ___: $7 + 5 = 12$ and $3(4) = 12$, therefore $7 + 5 = 3(4)$.

a. Transitive
b. Substitution
c. Multiplicative Property of Zero
d. Reflexive
e. Multiplicative Identity
f. Additive Identity
g. Symmetric
Properties Quiz

Instructions: Match each property with the problem that demonstrates it. (Each question is worth one point each.)

1. _g_: \[21 = 17 + 4, \text{ therefore } 4 + 17 = 21.\]

2. _d_: \[7 = 7\]

3. _f_: \[0 + 150 = 150\]

4. _c_: \[65 \times 0 = 0\]

5. _b_: \[15 - 6 = 9, \text{ therefore } 15 - 6 \text{ may be replaced by } 9.\]

6. _e_: \[2,000 \times 1 = 2,000\]

7. _a_: \[7 + 5 = 12 \text{ and } 3(4) = 12, \text{ therefore } 7 + 5 = 3(4).\]

a. Transitive
b. Substitution
c. Multiplicative Property of Zero
d. Reflexive
e. Multiplicative Identity
f. Additive Identity
g. Symmetric
LESSON PLAN by: Kyler Kearby

Lesson: Understanding logical reasoning

Length: 70 minutes Grade Intended: 9th and 10th

Academic Standard:
A1.9.6: Distinguish between inductive and deductive reasoning, identifying and providing examples of each.
A1.9.7: Identify the hypothesis and conclusion in a logical deduction.

Performance Objective: When the students are given five problems that involve identifying, applying, or distinguishing between inductive and deductive reasoning, they will correctly answer four out of the five problems.
When the students are given two problems that involve identifying the hypothesis and conclusion in a logical deduction, they will correctly identify the hypothesis and conclusion in both problems.

Assessment:
The students will be assigned five homework problems. For each problem, the student must identify, apply or distinguish between inductive and deductive reasoning to answer each problem. In addition, for two of these problems, the student must identify the hypothesis and conclusion of the given logical deduction. The teacher will evaluate the students on whether or not their answers are correct and whether or not the stated concepts have been correctly identified.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for students with behavioral disorders
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “Last class, we learned about the four-step strategy to solve problems. From the homework assignment, you were able to use this strategy to solve mathematical problems. Now, I would like you to use it to solve a problem that is not necessarily related to math.” Ask the students to get out a sheet of paper and have them explain how they would apply the four-step strategy to choose their classes for next semester/trimester. Give the students ten minutes to write their explanations and then collect them. They will not be collected as a grade as it is simply a check to see that the
students understand how to apply to four-step strategy to solve a given problem. Explain to the students that they have just done a form of today’s lesson—logical reasoning. Ask the students, “In your words, what is logical reasoning? Support you answer with an example of it.” (Bloom: Evaluation). Listen to a few students’ responses and conclude that logical reasoning is “when some type of strategy is used to solve a problem.” Say to the students, “You have definitely used logical reasoning in the past. Some of you have never realized it, though, as you were unaccustomed to the concept.” Tell the students that logical reasoning is highly needed in making any decision or solving any problem that they will come across in life. State that logical reasoning has two forms—inductive and deductive reasoning.

**Step-by-Step Plan:**

1. On the board, write “The first four numbers of a sequence are 1, 3, 6 and 10. List the next three numbers of this sequence.” Tell the students that the best way to solve this problem would be to find a pattern and then determine the next three numbers from this pattern. Ask the students, “What is the pattern in this sequence?” (Bloom: Analysis). If the students are unable to identify a correct pattern, explain to the students that the first number is added by 2 to get the next number, the second number is added by 3 to get the next number and the third number is added by 4 to get the next number. Ask the students, “With this in mind, what will the fourth number be added by to get the next number?” (Bloom: Application) Make sure that the students know it will be added by 5. Therefore, 10 + 5 = 15. Then, 15 will be added by 6 to get 21 and finally 21 will be added by 7 to get 28. So, the next three numbers of this sequence would be 15, 21 and 28. (Gardner: Logical/Mathematical and Interpersonal)

2. Tell the students that they have just completed a form of inductive reasoning. Define inductive reasoning as “where a conclusion is made based on a pattern or past events” and write this definition on the board. Tell the students that the other form of logical reasoning—deductive reasoning—can be defined as “the process of using facts, rules, definitions or properties in a logical order to reach a valid conclusion.” Be sure to write this definition on the board. Ask the students, “Based on these definitions, what is a difference between inductive and deductive reasoning?” (Bloom: Comprehension). Explain to the students that a main difference between the two forms of reasoning could be stated as inductive reasoning involves “perceived assumptions,” whereas deductive reasoning involves “known facts.” Make sure the students know that both forms share a common goal—to reach a conclusion. (Gardner: Logical/Mathematical, Interpersonal and Verbal-Linguistic)

3. Tell the students that a very common example of deductive reasoning is a “conditional” statement or rather, an “if-then statement.” Tell the students to think like scientists, instead of mathematicians, and ask them, “What do you know about the scientific method? Why do scientists use it? (Bloom: Analysis). Listen to their answers and tell them that the scientists use it to form conclusions from a stated hypothesis. Tell them that knowing the definitions of a hypothesis and a conclusion from science class will be very helpful in understanding and applying if-then statements in mathematics. Explain to them that an if-then statement is a sentence that contains a conclusion drawn from a hypothesis. The hypothesis is the portion of the sentence following “if,” and a conclusion is the part following “then.” Ask the students, “What are some examples of if-then statements?” (Bloom: Comprehension). Tell them whether or not their examples are correct. (Gardner: Logical/Mathematical, Interpersonal and Verbal-linguistic)

4. On the board, write the following conditional statement—“If I visit the city of Indianapolis, then I am in Indiana.” Below this statement, write the given statement—“I visit the city of Indianapolis.” Explain to the students that the goal is to determine whether or not this is a
valid statement. Ask the students, “Knowing that I visit the city of Indianapolis, am I in Indiana?” (Bloom: Application). The students should say yes as according to the conditional statement—“if you visit the city of Indianapolis, then you will be in Indiana.” Therefore, this is a valid conclusion. Erase the given statement and write “I am in Indiana.” Ask the students, “Why can a valid conclusion not be formed from this statement?” (Bloom: Evaluation). If the students are unable to correctly answer this question, explain to them that a valid conclusion cannot be formed as the conclusion has been given, instead of the hypothesis. Tell the students that valid conclusions can only be made when the hypothesis is given. (Gardner: Logical/Mathematical and Interpersonal)

5. Ask the students if they have any questions about the aspects of logical reasoning that they learned today. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them the “Extending the Investigation” activity (problems 1 – 5) on pg 31. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

Closure:
With a few minutes left in the class period, say to the students, “I hope you leave today having a decent understanding of the different forms of logical reasoning. When applying logical reasoning, you are developing critical-thinking skills. As teachers, our goal is to help you become critical thinkers and lifelong learners.” Tell the students that performing research and collecting data is one application of using critical-thinking skills. Ask them, “When you did research or collected data in the past, how did you collect it? Did you simply jot down the data, create tables or graphs, or something else?” (Bloom: Knowledge)

Adaptations/Enrichment:

Students with behavioral disorders:
1) Write class agenda on the board which will allow the students to know what is going to take place during class.
2) Allow students to have breaks if they become frustrated during the lesson. Let them use the restroom, run an errand, clean the chalkboard, etc.
3) Encourage and enforce a supportive classroom environment at all times.

Self-Reflection:
Do the students understand the difference between inductive and deductive reasoning?
Do the students understand the difference between a hypothesis and a conclusion?
Can the students identify examples of inductive and deductive reasoning?
Can the students identify the hypothesis and conclusion of an if-then statement?
Did I keep the students engaged throughout the entire lesson?
LESSON PLAN by: Kyler Kearby

Lesson: Collecting data using frequency tables

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
A1.9.1 Use a variety of problem solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Performance Objective: When the students are given four sets of data, they will correctly create a frequency table for three out of the four sets of data.

Assessment:
The students will be assigned a worksheet that includes four different sets of data. For each set of data, the student must create a frequency table to organize each set. The teacher will evaluate the students on the following criteria: 1) the student chooses accurate intervals, 2) the student records tally marks of the data and 3) the student lists the frequency of the tally marks.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for a student with autism
- Create and print off enough student copies of the “Collecting Data” worksheet (attached)
- Create and print off answer key of the “Collecting Data” worksheet (attached)
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “Last class, we focused on understanding logical reasoning and its two forms—deductive and inductive reasoning. Collecting data—which I asked you to think about at the end of the period—can be used in both forms of reasoning. Today, we will learn a way to collect data, and it would mainly be used in deductive reasoning.” Ask the students, “In the past, what types of information have you gathered and collected for?” (Bloom: Knowledge). Listen to their answers and ask them, “Why do people collect data anyways?” (Bloom: Knowledge). Explain to the students that the main purpose of collecting data is to answer questions whose answers are not immediately obvious. Tell them that collecting data can be used in forming conclusions and predictions about virtually anything such as the weather, money or a person’s health. Say to the students, “For example, one of my responsibilities as a teacher is to record your grades and attendance. From these records, I can make predictions on how often you will be in class
or what grade you will have at the end of the semester.” Tell the students that today’s lesson will focus on simply creating a way to collect data and eventually, they will be able to interpret this collected data.

**Step-by-Step Plan:**

1. Tell the students that one way to collect data is to organize it into a frequency table. In a frequency table, explain to the students that you use tally marks to record and display the frequency of events. Tell the students that frequency tables are divided into the following three columns: Title (of the given data), Tally and Frequency. Be sure to tell the students that underneath the “Title” column, accurate intervals pertaining to the given data are created. Ask the students, “With this in mind, what would be recorded under the “Tally” and “Frequency” columns?” (Bloom: Comprehension). If the students are unable to answer this question, explain to them that for the “Tally” column, they tally mark the data into the created intervals and then list the number of tallies in the “Frequency” column. (Gardner: Logical/Mathematical and Interpersonal)

2. Have each student write their height on the board. Explain to the students that they have just collected data, and now the objective is to organize this data as a frequency table. On the board, create the three columns of a frequency table. Be sure to label the “Title” column as “Heights.” Ask the students, “What are some intervals that could be used to record these heights?” (Bloom: Analysis). (If the heights arranged from 4’11” to 6’2”, then accurate intervals would be 5’11” to 6’2”, 5’7” to 5’10”, 5’3” to 5’6” and 4’11” to 5’2”). Place these intervals under the “Heights” column. Next, record tally marks for the data that falls into these intervals. Count the number of tallies for each interval and place these numbers under the “Frequency” column. Lastly, identify a title for the overall table such as “Students’ Heights.” (Without the recorded tally marks and their frequencies, the final table should resemble Table 1.) (Gardner: Logical/Mathematical, Bodily-Kinesthetic, Interpersonal and Visual-Spatial)

<table>
<thead>
<tr>
<th>Heights (ft)</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5’11” – 6’2”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5’7” – 5’10”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5’3” – 5’6”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4’11” – 5’2”</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Write the following high temperatures (°F) of different states on the board: 51, 68, 77, 61, 55, 57, 72, 79, 59, 63, 74, 56, 64, 73, 76, 58, 52, 72, 76 and 68. Draw the components of a frequency table on the board and label the first column as “Temperatures.” Ask the students, “What could be some accurate intervals of these temperatures?” (Bloom: Analysis). (One possibility could consist of the following intervals: 51 – 55, 56 – 60, 61 – 65, 66 – 70, 71 – 75 and 76 – 80.) Have different students come up to the board and place tally marks in the different intervals. Once
the tally marks have been recorded, count them and place these numbers under the “Frequency” column. Lastly, remind the students to give the table a title such as “High Temperatures of Varied States.” (The final table should resemble Table 2.) (Gardner: Logical/Mathematical, Bodily-Kinesthetic, Interpersonal and Visual-Spatial)

Table 2

<table>
<thead>
<tr>
<th>Temperatures (°F)</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>51 – 55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56 – 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>61 – 65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>66 – 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71 – 75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>76 – 80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Ask the students if they have any questions about collecting data using frequency tables. If they have questions, answer them. Otherwise, pass out the “Collecting Data” worksheet to the students. Let them work on the worksheet for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

Closure:
With a few minutes left in the class period, say to the students, “After today, you should have a good understanding of how to collect data using frequency tables. This is definitely not the only way to collect data, though, as there are a variety of ways to record data.” Explain to the students that the next step will be to learn how to display collected data. Ask them, “What are some ways to display data? In your opinion, which way would you prefer to use and why?” (Bloom: Evaluation)

Adaptations/Enrichment:

Student with autism:
1) Use reinforcers specific to the needs of the student and allow the student to work with a partner.
2) Warn the student about transitions and be sure to show the student exactly what you would like him or her to do as far as behaving, listening, working on the assignment and so forth.
3) Incorporate many visuals—such as the frequency tables themselves—into the lesson and create a learning environment that makes the student feel comfortable.

Self-Reflection:
Do the students understand the importance of collecting data?
Do the students understand how to collect data using frequency tables?
Are the students willing to participate during classroom discussions?
Did I keep the students engaged throughout the entire lesson?
Collecting data

Instructions: Make a frequency table to organize each set of data.

15. For each game, a soccer team scored the following number of goals: 1, 2, 5, 1, 6, 2, 6, 8, 4, 2, 4, 5, 5, 1, 3, 4, 7, 2, 2, 6 and 4.

16. A teacher recorded the following test scores: 84, 67, 89, 98, 77, 84, 92, 64, 79, 81, 95, 73, 66, 59, 96, 88, 71, 68, 90 and 86.


Collecting data

Instructions: Make a frequency table to organize each set of data.

1. For each game, a soccer team scored the following number of goals: 1, 2, 5, 1, 6, 2, 6, 8, 4, 2, 4, 5, 5, 1, 3, 4, 7, 2, 2, 6 and 4.

<table>
<thead>
<tr>
<th>Number of Goals</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2</td>
<td>¶¶¶</td>
<td></td>
</tr>
<tr>
<td>3 – 4</td>
<td>¶ ¶</td>
<td>5</td>
</tr>
<tr>
<td>5 – 6</td>
<td>¶ ¶</td>
<td>6</td>
</tr>
<tr>
<td>7 – 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Interval may vary.)

2. A teacher recorded the following test scores: 84, 67, 89, 98, 77, 84, 92, 64, 79, 81, 95, 73, 66, 59, 96, 88, 71, 68, 90 and 86.

<table>
<thead>
<tr>
<th>Scores</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 – 99</td>
<td>¶ ¶ ¶</td>
<td>5</td>
</tr>
<tr>
<td>80 – 89</td>
<td>¶ ¶ ¶</td>
<td>6</td>
</tr>
<tr>
<td>70 – 79</td>
<td>¶ ¶ ¶ ¶</td>
<td>4</td>
</tr>
<tr>
<td>60 – 69</td>
<td>¶ ¶ ¶ ¶</td>
<td>4</td>
</tr>
<tr>
<td>59 – below</td>
<td>¶</td>
<td>1</td>
</tr>
</tbody>
</table>

(Interval may vary.)


<table>
<thead>
<tr>
<th>Topping</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheese</td>
<td>¶ ¶ ¶</td>
<td>5</td>
</tr>
<tr>
<td>Pepperoni</td>
<td>¶ ¶ ¶</td>
<td>5</td>
</tr>
<tr>
<td>Sausage</td>
<td>¶ ¶ ¶</td>
<td>3</td>
</tr>
<tr>
<td>Vegetarian</td>
<td>¶ ¶ ¶</td>
<td>2</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adventure</td>
<td>¶ ¶ ¶ ¶</td>
<td>7</td>
</tr>
<tr>
<td>Comedy</td>
<td>¶ ¶ ¶ ¶ ¶</td>
<td>9</td>
</tr>
<tr>
<td>Drama</td>
<td>¶ ¶ ¶</td>
<td>3</td>
</tr>
<tr>
<td>Horror</td>
<td>¶ ¶ ¶</td>
<td>2</td>
</tr>
</tbody>
</table>
L. LESSON PLAN MODIFIED FOR INTELLECTUAL DISABILITY

MANCHESTER COLLEGE
Department of Education

LESSON PLAN by: Kyler Kearby

Lesson: Displaying data using line graphs, bar graphs and histograms

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
A1.9.1 Use a variety of problem solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Performance Objective:
When the students are given four collections of data, they will correctly create a line graph, bar graph, or histogram for three out of the four collections of data.

Assessment:
The students will be assigned four homework problems. For each problem, the student must create a line graph, bar graph, or histogram depending on what the problem asks for. The teacher will evaluate the students on the following criteria: 1) the student chooses accurate intervals, 2) the student labels each axis and titles the graph and 3) the student accurately plots the points or draws the bars.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for a student with an intellectual disability
- Copy of answer key for the “Collecting Data” worksheet (previous assignment)
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “At the end of last class, I asked you to think about some different types of graphs or charts that can be used to display collections of data. Today, we are going to focus on these different types of displays—mainly line graphs, bar graphs and histograms—which hopefully will be a review day for most of you.” Ask the students, “Why do people create graphs and charts? I mean, the data has previously been collected, so why do people still need to display the collected data?” (Bloom: Comprehension). Explain to the students that graphs and charts organize the collected data into visual-aids that allow people to easily see and interpret the information. Researchers use graphs to help them make predictions on whatever it is they are researching. Tell the students that they could make a graph to analyze how they have done on their tests throughout the semester.
Step-by-Step Plan:

1. Ask the students, “Based on prior knowledge, what is the difference between a line graph and a bar graph or histogram?” (Bloom: Knowledge). Make sure the students know that line graphs show trends or changes over time, whereas bar graphs and histograms show the frequency of items in sets of data. Ask the students, “Thinking back to when you first learned about the different types of graphs or charts, can you recall what the main difference is between a bar graph and a histogram?” (Bloom: Knowledge). Tell the students that the main difference is that bar graphs contain spaces between the bars, whereas histograms do not. Remind the students that all graphs must have a title, labels for each axis and also equal intervals for each axis. (Gardner: Logical/Mathematical and Interpersonal)

2. On the board, draw a table that includes the following information about the approximate consumption of bottled water per Indiana resident: 1997 (9 gal), 1999 (10.5 gal), 2001 (12 gal), 2003 (14 gal), 2005 (17 gal) and 2007 (19.5 gal). (The table should resemble the table on the right). Ask the students, “Looking at this table, what type of graph should we create to display this information? Why?” (Bloom: Analysis). A line graph should be created as this data represents changes over a period of time. Draw a horizontal and vertical axis on the board. Ask the students about ideas for a title of the graph and also labels for each axis. Have a student write accurate intervals on the board, plot the data and draw a line connecting the points of data. (The line graph should resemble the graph below.) (Gardner: Logical/Mathematical, Bodily-Kinesthetic, Interpersonal and Visual-Spatial)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bottled Water (gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>9</td>
</tr>
<tr>
<td>1999</td>
<td>10.5</td>
</tr>
<tr>
<td>2001</td>
<td>12</td>
</tr>
<tr>
<td>2003</td>
<td>14</td>
</tr>
<tr>
<td>2005</td>
<td>17</td>
</tr>
<tr>
<td>2007</td>
<td>19.5</td>
</tr>
</tbody>
</table>

3. Take out the answer key for the “Collecting Data” worksheet and draw the table for problem 2 on the board. Tell the students that this table represents the collected data for problem 2 from the previous day’s worksheet. Ask the students, “What type of graph would be the best representation of this data?” (Bloom: Analysis). The students should say either a bar graph or histogram—make a bar graph. Tell the students that the title, labels and intervals of the table should be kept the same for the bar graph. Draw a horizontal and vertical axis on the board. Have a student come to the board and create the bars representing the data. (The bar graph should resemble the graph below.) Ask the students, “What would we need to do in order to turn this bar graph into a histogram?” (Bloom: Comprehension). The space between each bar would need to be removed. (Gardner: Logical/Mathematical, Bodily-Kinesthetic, Interpersonal and Visual-Spatial)
4. Ask the students if they have any questions displaying data using line graphs, bar graphs and histograms. If they have questions, answer them. Otherwise, have the students get their textbooks out and assign them problems (4, 9, 12 and 15) on pgs. 42 – 43. Let the students work on this assignment for the rest of the class period. Walk around the room and answer questions the students may have. (Gardner: Logical/Mathematical and Interpersonal)

**Closure:**
With about ten minutes left in the class period, tell the students that the chapter test will be two days from now. Then, say to the students, “I hope you now feel very confident in displaying data using line graphs, bar graphs and histograms.” Ask the students, “What are the differences between these graphs and what would be some circumstances in which you would need to use each of these graphs? (Bloom: Comprehension). Tell the students that as their ticket out of class they need to answer this question on a separate sheet of paper and then turn it in. Before the bell rings, ask them, “After researchers make graphs, they form conclusions. However, their conclusions are not always true. For these occasions, how could we prove them wrong?” Bloom: Synthesis).

**Adaptations/Enrichment:**

**Student with an intellectual disability:**

1) Make sure the students most importantly understand how to create the graphs. After this, explain to the students that each graph is used to display a certain type of information. Help them choose which graphs to display the given data.

2) Reduce the assignment—give the student only two problems to do, instead of four.

3) Provide the student with a peer tutor. Have the peer tutor help the student choose which type of graph to use to display the given data and help create the chosen graph.

**Self-Reflection:**
Do the students understand the differences between line graphs, bar graphs and histograms?
Do the students understand how to display data using line graphs, bar graphs and histograms?
Did I keep the students engaged throughout the entire lesson?
Were the students willing to participate in classroom discussions?
M. LESSON PLAN MODIFIED FOR SENSORY IMPAIRMENT

MANCHESTER COLLEGE
Department of Education

LESSON PLAN by: Kyler Kearby

Lesson: Making counterexamples

Length: 70 minutes

Grade Intended: 9th and 10th

Academic Standard:
A1.9.8 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Performance Objective:
When the students are given four statements, they will give a counterexample for three out of the four statements.

Assessment:
The students will be given a worksheet that contains four graphs and a statement relating to each graph. For each statement, the student must give a counterexample to show that the statement is false. The teacher will evaluate the students simply on whether or not they have given counterexamples to falsify the statements.

Advanced Preparation by Teacher:
- Review lesson plan and make sure it has been modified for a student with a sensory impairment
- Create and print off enough student copies of the “Differences of Graphs” handout (attached)
- Create and print off teacher copy of the “Differences of Graphs” handout (attached)
- Create and print off enough student copies of the “Why is the statement false?” worksheet (attached)
- Create and print off answer key of the “Why is the statement false?” worksheet (attached)
- Clean chalkboard
- Fresh chalk

Procedure:

Introduction/Motivation:
First, say to the students, “At the end of our last class, each of you handed in a piece of paper that compared the different types of graphs. To make these differences easier to see, I have created a matrix that covers these differences. Right now, I would like for you to complete individually.” Pass out the “Differences of Graphs” handout and give the students about ten minutes to complete it. Walk around the room and make sure the students are filling it out correctly. Say to the students, “Once the data has been displayed, researchers and scientists can start drawing conclusions. However, their conclusions can sometimes be false.” Ask the students, “For example, let’s think about commercials. Should we believe everything that commercials or advertisers tell us? Why or why not?” (Bloom: Evaluation). Tell the students that people cannot
prove that conclusions or statements are false without evidence. With this in mind, it is important that people know how to provide counterexamples to falsify conclusions or statements.

**Step-by-Step Plan:**

1. Ask the students, “What is a counterexample?” (Bloom: Knowledge). Make sure the students know that it is an example or statement that contradicts a previous conclusion. On the board, draw the finished graph from the previous lesson’s first example. (It should resemble the graph on the right.) Below the graph, write the statement—“The least amount of bottled water consumed per resident was in 1999.” Ask the students, “Why is the statement false?” (Bloom: Analysis). It is false because as there exists a counterexample—1997 was the year when the least amount of bottled water was consumed per resident. Tell the students that they only need to find one counterexample to prove a statement or conclusion wrong. (Gardner: Logical/Mathematical, Interpersonal and Visual-Spatial)

2. On the board, draw the finished graph from the previous lesson’s second example. (It should resemble the graph on the right.) Below the graph, write the statement—“There were more scores in the 70 – 79 range than there were in the 60 – 69 range.” Ask the students, “Does there exist a counterexample to this statement? If so, what is it?” (Bloom: Application). The counterexample would be that there four people had scores in the 70 – 79 range and the same number of people had scores in the 60 – 69 range. With this in mind, there was an equal amount of scores in the 70 – 79 and 60 – 69 ranges. (Gardner: Logical/Mathematical, Interpersonal and Visual-Spatial)

3. Ask the students if they have any questions about making counterexamples to prove statements false. Pass out the “Why is the statement false?” worksheet to each student. Walk around the room and answer any questions the students may have. Once the students have completed their worksheets, go over the worksheet with the students as a class. Then, collect the worksheets. (Gardner: Logical/Mathematical and Interpersonal)

4. With the remaining time left, ask the students if they have any questions concerning the test. Answer any questions the students have. (Gardner: Logical/Mathematical and Interpersonal)
Closure:
Before the bell rings, say to the students, “Today concluded our chapter over learning the language of Algebra. I think each of us have a good understanding of this language now. Our next chapter will focus on integers.” As the students walk out the door, remind them to study for the test.

Adaptations/Enrichment:

Student with a sensory impairment:

Vision impairment:

1) Allow the student to wear a hat or visor if they have sensitivity to light and seat the student away from the window.
2) Provide the student with dark lead pencils, boldly lined paper or a magnifying glass.

Hearing impairment:

3) Sit the student around the area in which the teacher use speaks and provide the student with questions or handouts.
4) Repeat information and use visuals aids such as PowerPoint or visual instructions plans.

Self-Reflection:
Do the students understand how to make counterexamples to prove statements false?
Did I keep the students engaged throughout the entire lesson?
Do the students like using graphic organizers?
Do the students understand the “Language of Algebra?”
<table>
<thead>
<tr>
<th>Type of Graph</th>
<th>Line Graph</th>
<th>Bar Graph</th>
<th>Histogram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What does it look like?</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>What does it show?</strong> (Definition)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>What data could it be used to show?</strong> (Give an example.)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Differences of Graphs

<table>
<thead>
<tr>
<th>Type of Graph</th>
<th>Line Graph</th>
<th>Bar Graph</th>
<th>Histogram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What does it look like?</strong></td>
<td><img src="image1.png" alt="Line Graph" /></td>
<td><img src="image2.png" alt="Bar Graph" /></td>
<td><img src="image3.png" alt="Histogram" /></td>
</tr>
<tr>
<td><strong>What does it show?</strong> (Definition)</td>
<td>Trends or changes over time</td>
<td>The frequency of items in sets of data</td>
<td>The frequency of items in sets of data</td>
</tr>
<tr>
<td><strong>What data could it be used to show?</strong> (Give an example.)</td>
<td>Temperatures throughout a given week, gas prices over the past 10 years, cost of tuition while in college, etc.</td>
<td>Test scores, what type of candy bar a population likes, number of goals scored, etc.</td>
<td>Test scores, what type of candy bar a population likes, number of goals scored, etc.</td>
</tr>
</tbody>
</table>
Why is the statement false?

Instructions: For each statement, provide a counterexample to prove the statement false.

1. **Statement:** The Grand Canyon had the largest amount of visitors in 1990.
   **Counterexample:**

   ![Grand Canyon Visitors graph]

2. **Statement:** There were just as many kids run the 200 m dash between 15 and 24 seconds than between 35 and 44 seconds.
   **Counterexample:**

   ![200 Meter Dash bar graph]
3. **Statement:** The age group of 61 – 70 had the largest amount of participants for the survey.
**Counterexample:**

4. **Statement:** There was a higher percentage of Indiana homes with access to the Internet in 1999 than in 2000.
**Counterexample:**
Name: ____________________  Date: ________  Period: ___

Why is the statement false?

Instructions: For each statement, provide a counterexample to prove the statement false.

1.

Grand Canyon Visitors

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People (millions)</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Statement: The Grand Canyon had the largest amount of visitors in 1990.

Counterexample:
Based on the information in the graph, the Grand Canyon had the largest amount of visitors in 2000.

2.

200 Meter Dash

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>15 - 24</th>
<th>25 - 34</th>
<th>35 - 44</th>
<th>45 - 54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Kids</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Statement: There were just as many kids run the 200 m dash between 15 and 24 seconds than between 35 and 44 seconds.

Counterexample:
Based on the information in the graph, eight kids ran the 200 m dash between 15 and 24 seconds, whereas seven kids ran it between 35 and 44 seconds. Therefore, more kids ran the 200 m dash between 15 and 24 seconds than between 35 and 44 seconds.
3. Statement: The age group of 61 – 70 had the largest amount of participants for the survey.
   Counterexample: Based on the information in the graph, the age group of 31 – 40 had the largest amount of participants for the survey.

4. Statement: There was a higher percentage of Indiana homes with access to the Internet in 1999 than in 2000.
   Counterexample: Based on the information in the graph, the percentage of Indiana homes with access to the Internet was higher in 2000 than in 1999. Therefore, there was a higher percentage of Indiana homes with access to the Internet in 2000 than in 1999.
Chapter 1 Test

Write an algebraic expression for each verbal expression.

1. the difference of 1 and n
2. seven lesson than the quotient of j and p

Find the value of each expression.
3. 42 – (24 ÷ 2) + 10
4. 10(6 – 5) – (20 ÷ 2)

Simplify each expression.
5. 17 + k + 23
6. 2 · j · 7
7. 5(n + 2r) + 3n

For each logical deduction, underline the hypothesis and circle the conclusion.
8. If I am in Ohio, then I am in the United States.
9. If Joey walks to school, then he will be late.

Match each property with the problem that demonstrates it.
10. ____ : 7(3 + 8) = (7 × 3) + (7 × 8)
a. Transitive
b. Distributive Property
c. Commutative Property of Multiplication
d. Reflexive
e. Associative Property of Addition
f. Symmetric
11. ____ : If 4 + 6 = 10 and 10 = 2(5), then 4 + 6 = 2(5)
12. ____ : 70 = 70
d. Reflexive
e. Associative Property of Addition
f. Symmetric
13. ____ : 8 × 3 = 3 × 8
c. Commutative Property of Multiplication
14. ____ : (16 + 9) + 6 = 16 + (9 + 6)
d. Reflexive
e. Associative Property of Addition
f. Symmetric
15. ____ : If 18 = 5 + 13, then 5 + 13 = 18
d. Reflexive
e. Associative Property of Addition
f. Symmetric

16. Explain the difference between inductive and deductive reasoning. Be sure to provide an example of each type.
17. Over the past ten years, the enrollments of first-years at a college are as followed: 543, 565, 549, 533, 554, 561, 543, 539, 548 and 552. **Use the four-step plan to predict the enrollment of first-years for next year.** Include a frequency table of this data and a graph that would be the most suitable to display this data.

18. For the statement, provide a counterexample to prove the statement false.

**Statement:** The age group of 30 – 34 had the largest amount of participants for the survey.

**Counterexample:**
Chapter 1 Test (Answer Key)

Write an algebraic expression for each verbal expression.

1. the difference of 1 and n

\[ n - 1 \]

2. seven less than the quotient of j and p

\[ \frac{j}{p} - 7 \]

Find the value of each expression.

3. \[ 42 - (24 ÷ 2) + 10 \]

\[ 42 - 12 + 10 = 30 \]

4. \[ 10(6 - 5) - (20 ÷ 2) \]

\[ 10(1) - 10 = 0 \]

Simplify each expression.

5. \[ 17 + k + 23 \]

\[ k + 40 \]

6. \[ 2 \cdot j \cdot 7 \]

\[ j \cdot 14 \]

7. \[ 5(n + 2r) + 3n \]

\[ 5n + 10x + 3n = 8n + 10x \]

For each logical deduction, underline the hypothesis and circle the conclusion.

8. If I am in Ohio, then I am in the United States.

9. If Joey walks to school, then he will be late.

Match each property with the problem that demonstrates it.

10. \[ b \] : \[ 7(3 + 8) = (7 \times 3) + (7 \times 8) \]

   a. Transitive

11. \[ a \] : If \[ 4 + 6 = 10 \] and \[ 10 = 2(5) \], then \[ 4 + 6 = 2(5) \]

   b. Distributive Property

12. \[ d \] : \[ 70 = 70 \]

   c. Commutative Property of Multiplication

13. \[ c \] : \[ 8 \times 3 = 3 \times 8 \]

   d. Reflexive

14. \[ e \] : \[ (16 + 9) + 6 = 16 + (9 + 6) \]

   e. Associative Property of Addition

15. \[ f \] : If \[ 18 = 5 + 13 \], then \[ 5 + 13 = 18 \]

   f. Symmetric
16. Explain the difference between inductive and deductive reasoning. Be sure to provide an example of each type.

Sample Answer: The difference between inductive and deductive reasoning is that inductive reasoning involves perceived assumptions, whereas deductive reasoning involves known facts. An example of inductive reasoning would be finding numbers of a sequence, while an example of deductive reasoning would be an if-then statement.

17. Over the past ten years, the enrollments of first-years at a college are as followed: 543, 565, 549, 533, 554, 561, 543, 539, 548 and 552. Use the four-step plan to predict the enrollment of first-years for next year. Include a frequency table of this data and a graph that would be the most suitable to display this data. (Your prediction does not have to be exact. It simply needs to be an estimated number.

Sample Answer:

Explore: I need to predict the enrollment of first-years for next year at this college. The enrollment of first-years over the past ten years has been given to me

Plan: I am going to collect the given data using a frequency table and then, make a bar graph to best display this collected data. After reviewing this data, I will make a prediction on how many first-years will attend this college next year.

Solve:

<table>
<thead>
<tr>
<th>Enrollment of First-years</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>530 – 537</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>538 – 545</td>
<td></td>
<td></td>
</tr>
<tr>
<td>546 – 553</td>
<td></td>
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<tr>
<td>554 – 561</td>
<td></td>
<td></td>
</tr>
<tr>
<td>562 – 569</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
From this data, I predict that next year’s enrollment will be in the range of 548. (I will accept any answer between 546 and 550.)

**Examine:** Based on the frequency table and graph, I feel that 548 is a pretty good prediction how many first-years will attend this college next year.

18. For the statement, provide a counterexample to prove the statement false.

**Statement:** The age group of 30 – 34 had the largest amount of participants for the survey.

**Counterexample:**

**Sample Answer:** Based on the information in the graph, the age group of 35 – 39 had the largest amount of participants for the survey.
Chapter 1 Test

Write an algebraic expression for each verbal expression.

1. the difference of 1 and n
2. seven lesson than the quotient of j and p

Find the value of each expression.

3. $42 - (24 ÷ 2) + 10$
4. $10(6 - 5) - (20 ÷ 2)$

Simplify each expression.

5. $17 + k + 23$
6. $5(n + 2r) + 3n$

Match each property with the problem that demonstrates it.

7. ___: $7(3 + 8) = (7 \times 3) + (7 \times 8)$
   a. Transitive
   b. Distributive Property
   c. Commutative Property of Multiplication
   d. Reflexive

8. ___: If $4 + 6 = 10$ and $10 = 2(5)$, then $4 + 6 = 2(5)$

9. ___: $70 = 70$

10. ___: $8 \times 3 = 3 \times 8$

11. Define inductive reasoning and deductive reasoning. Provide an example of each type of reasoning.
For each logical deduction, underline the hypothesis and circle the conclusion.

12. If I am in Ohio, then I am in the United States.

13. If Joey walks to school, then he will be late.

14. Over the past ten years, the enrollments of first-years at a college are as followed: 543, 565, 549, 533, 554, 561, 543, 539, 548 and 552. Use the four-step plan to predict the enrollment of first-years for next year. Include a frequency table of this data and a bar graph to display the data.
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**Counterexample:**
Chapter 1 Test (Answer Key)

Write an algebraic expression for each verbal expression.

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   \[ n - 1 \]
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   \[ \frac{j}{p} - 7 \]

Find the value of each expression.

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   \[ k + 40 \]
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   \[ 8n + 10x \]

Match each property with the problem that demonstrates it.

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   a. Transitive
   b. Distributive Property
   c. Commutative Property of Multiplication
   d. Reflexive
8. \_a_: If \[ 4 + 6 = 10 \] and \[ 10 = 2(5) \], then \[ 4 + 6 = 2(5) \]
9. \_d_: \[ 70 = 70 \]
10. \_c_: \[ 8 \times 3 = 3 \times 8 \]

11. Define inductive reasoning and deductive reasoning. Provide an example of each type of reasoning.

Sample Answer: Inductive reasoning is where a conclusion is made based on a pattern or past event, and an example of inductive reasoning would be finding numbers of a sequence. Deductive reasoning is the process of using facts, rules, definitions, or properties in a logical order to reach a valid conclusion, and an example of deductive reasoning would be an if-then statement.
For each logical deduction, underline the hypothesis and circle the conclusion.

12. If I am in Ohio, then I am in the United States.

13. If Joey walks to school, then he will be late.

14. Over the past ten years, the enrollments of first-years at a college are as followed: 543, 565, 549, 533, 554, 561, 543, 539, 548 and 552. Use the four-step plan to predict the enrollment of first-years for next year. Include a frequency table of this data and a bar graph to display the data.

Sample Answer:

Explore: I need to predict the enrollment of first-years for next year at this college. The enrollment of first-years over the past ten years has been given to me

Plan: I am going to collect the given data using a frequency table and then, make a bar graph to best display this collected data. After reviewing this data, I will make a prediction on how many first-years will attend this college next year.

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Examine: Based on the frequency table and graph, I feel that 548 is a pretty good prediction how many first-years will attend this college next year.

15. For the statement, provide a counterexample to prove the statement false.

Statement: The age group of 30 – 34 had the largest amount of participants for the survey.

Counterexample:

Sample Answer: Based on the information in the graph, the age group of 35 – 39 had the largest amount of participants for the survey.
**Modification Explanation:** The modified test has the following modifications: provides more space for the students to answer questions, reduces the amount of questions, re-wording of some questions and puts the questions in order according to when they were taught. Many students with learning disabilities have poor organizational skills and providing more space will allow them to show their work and answers in a more organized way. By reducing a few questions, the students will have less to process and understand before computing or stating their answers. This will help them complete the test and reduce test anxiety while still working just as hard as the rest of the students. In order for the students to know exactly what I want in their answers, I restated some of the instructions. For example, instead of asking the students to differentiate between deductive and inductive reasoning, I simply asked them to state the definition of each. The students are still stating the same answer, but the question is asked in a more applicable way for them to understand. For the question in which I asked the students to create a graph, I told the students to create a bar graph, instead of having them choose the graph that would be best suited to display the given data. The students do not have to take time to think about which graph to display as they can just create the bar graph. Lastly, I put the questions in the order in which they were taught. This way the students will not be surprised with the order of the questions as they are already familiar with it.

During the test, additional accommodations can be made. One modification would be to give the students additional time to complete their test to accommodate for their slower rate of processing. If the students struggle with basic computations, they could be allowed to use a calculator. Another modification would be to have the students complete the test in an isolated place to reduce distractions. A final modification would be to have someone read the test aloud to support their learning style and individual needs.
Throughout my life, I have never been extremely passionate about reading. I mean, I am not the type of person who finds it relaxing to sit on the couch and read a good book. However, I still know the importance of being able to read. Reading gives people the knowledge and power to develop into competent citizens. Our communication system relies mainly on spoken and written communication and for this reason, the ability to read is a must. In order to understand spoken communication, some people can get away without being able to read as they simply need to be able listen to and comprehend what the person is saying. On the other hand, in order to understand written communication, a person must have the ability to read. If this person does not know how to read, he or she will not be able to interpret anything written. From this, people who do not know how to read will have a very difficult time in just trying to survive. Based on this fact alone, everyone needs to have the ability to read and understand written language.

From this course, I learned that I will not be the only teacher in my classroom as there will be plenty of silent teachers in my classroom—teachers who say plenty without ever speaking a word. These silent teachers consist of books, magazines, newspapers, graphic organizers, visual aids and so forth. Although they never say a word, they can have just as much, or even more of an impact as I have on the students. I mean, books have the ability to get students interested to a lesson or concept; visual aids, handouts, graphic organizers, etc. have the ability to instruct a classroom full of students; and all of these silent teachers have the ability to keep the students engaged. With this in mind, I need to incorporate these nonhuman teachers into my lessons because they can serve as the students’ teacher whenever I am not around such as when I am working with a student one-on-one or when the students are at home.
The one tool that I think I will utilize the most in my classroom consists of using trade books. Most math textbooks provide the students with good examples of how to do the problems, but they do not put mathematical concepts into readings. On the other hand, trade books can and do achieve this desired goal. Trade books simply put math into writing and allow students to understand mathematics through reading. I am not putting down my content area, but some mathematical concepts can be boring. In these few situations, I will do my best to make these concepts exciting on my own, but I can also utilize trade books for support. Trade books have the ability to make for a fun and an enjoying read and therefore, can make these “boring” math concepts stimulating. Most importantly, though, trade books demonstrate how mathematics can be used in real-life situations—which will show the students when they can use math and why they need to understand math.

I understand that students definitely need to have the ability to read and therefore, I will do my best to incorporate reading into my classroom. Many math teachers focus only on teaching their students how to compute problems and solve equations. They neglect putting mathematics into reading and writing, which gives students the idea that there does not exist a connection between these content areas. However, there does exist a connection between reading, writing and mathematics, and I will demonstrate this connection through the use of trade books, writing to learn strategies and graphic organizers.