Lesson Plan by Aaron Cripe

Lesson: Permutations and Combinations Chapter 4 Section 3

Length: 70 minutes

Grade: Discrete Mathematics

Academic Standards:

Standard 1 — Counting Techniques
Students develop an understanding of combinatorial reasoning, using various types of diagrams and the fundamental counting principle to find numbers of outcomes and related probabilities. They also use simulations to solve counting and probability problems.

DM.1.2 - Use the fundamental counting principle to find the number of outcomes in a problem situation.

Performance Objectives: After today's lesson the students will solve problems by applying permutations and combinations with 80% accuracy on the daily assessment homework assignment. Permutations and combinations will also be on a paper and pencil test that will be assessed at the end of the chapter. Students should also recognize when to use permutations or combinations instead of the other counting techniques when solving a problem.

Assessment: When the students walk into the classroom there will be a daily assessment or homework assignment written on the board. This assignment will be due at the beginning of the next class. The goal of the daily assessment is to give extra practice and test the knowledge and understanding of the lesson of that day. In the homework assignment the students will be applying permutations and combinations to solve various problems. This will help give the teacher feedback to where the students are at. There will also be a paper and pencil assessment at the end of the chapter, and a quiz in the middle of the chapter to help test the students overall knowledge of the lessons.
Homework Assignment: Page 324-325 (1-19 odd)

Advanced Preparation by Teacher: The teacher should be prepared to teach the lesson and have all the accommodations ready for the gifted and talented students. The teacher should have the agenda, homework assignment, and bell ringer ready when students come to class. The teacher should already have the groups divided out on paper for when the teacher chooses groups. The teacher should have all the examples ready when students come to class, with appropriate solutions.

Procedure:

Introduction: When the students come to class the teacher should have already prepared a bell ringer activity that the students should begin when they arrive. The bell ringer should involve the pigeonhole principle since that is what the last lesson's assignment covered.

Bell Ringer:
A bowl contains ten red balls and ten blue balls. A woman selects balls at random without looking at them.

1) How many balls must she select to be sure of having at least three balls of the same color?
2) How many balls must she select to be sure of having at least three blue balls?

It is good to provide a little review before you start a new lesson. The agenda for the day should also be written on the board for the students when they come in. While the students are working on the bell ringer activity the teacher should use the time to take attendance and make any further preparations for the lesson. When the students have finished the bell ringer activity it is important for the teacher to go over that activity. Make sure that all of the students are on the same page before you begin the new lesson. Now it is time to start today's lesson. The teacher should begin with an example on the board.

<u>Example on Board</u>
If the teacher wants to choose 5 students from the class today (lets say 20 students are in class today), how many different possible options would the teacher have for his/her decision?
(Note: it does not matter the order or who gets picked)

The teacher should then have the students start writing down possible options, and have them try to figure out the answer on their own. The teacher knows that the answer is way too large for the students to be able to write down all the possibilities, but the teacher should let the students try anyways for about 2 to 3 minutes. After the time is up the teacher should see students who are frustrated, because they have realized how many possible solutions there are. The teacher should ask the students what the highest number they came up with was, and then the teacher should tell them the answer is 15,504. Teacher should explain to the students that in today's lesson they will learn a quick and easy way to find that answer.

Step by Step Procedure: Teacher should begin by explaining that today's lesson will be over permutations and combinations. Then teacher should explain exactly what each one of those words means. A permutation is a set of distinct objects is an **ordered** arrangement of these objects. A combination is a set of **unordered** elements from the total set. The teacher should ask the students if they see the difference between the two definitions (Bloom's Level of Knowledge). Then the teacher should ask the students what they think this means, and how it affects the problem (Bloom's Level of Comprehension). The teacher should then give a simple example of a permutation problem followed by a simple example of a combination problem (no need to solve yet).
Example of Permutation:
How many ways are there to select a first prize winner, second-prize winner, and a third-prize winner from 5 different people who have entered a contest?

Example of a Combination:
How many sets can you create from the set = (a, b, c, d) when each set has to have 2 elements in it?

The teacher should then explain the difference between the two problems and why the first is a permutation and the second a combination. Once the teacher feels that the class understands the difference between the two, then it is time to move on to formulas.

The formula for permutations is n! / (n-r)!, where n is the number of distinct elements and r is number of permutations in the set. The teacher should then go over some examples and teach the students how to apply the formula. Then the teacher should have the students do some examples by themselves at their desk (Multiple Level of Intrapersonal).

<u>Example 1</u>
Suppose that there are eight runners in a race. The winner receives a gold medal the second-place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?
Solution: Since there are 8 runners and 3 medals
P(8,3) = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 / 5 * 4 * 3 * 2 * 1 = 8 * 7 * 6 = 336
The teacher should make sure to write this whole solution out on the board and explain step by step what gets crossed off and why you can simplify (Multiple Level of Spatial).

<u>Example 2</u>
Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?
Solution: The first city is locked in but the other seven can be chosen however she wants them to be. Thus the answer is 7! = 5040

<u>Example 3 for Students at their desks</u>
How many permutations of the letters ABCDEFGH contain the string ABC?
Solution: Since we have a block of 3 that have to occur, the other six letters can be in any order, thus the answer is 6! = 720.

Now the teacher should do the same with combinations. The teacher should explain the combinations formula, which is:
n! / r! (n-r)! where r is the number of combinations and n is the number of elements. The teacher should then go over examples as a class then have the students do one example by themselves.
<u>Example 1 (Multiple Level of Logical)</u>
How many way are there to select five player from a 10-member tennis team to make a trip to a match at another school?
Solution:
Using the formula you can set it up C(10, 5) = 10! / 5! * 5! = 252

Example 2
A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?
Solution:
Using the formula C(30, 6) = 30! / 6! * 24! = 593,775

Example 3 for students to do at their desks (Bloom's Level of Application)
How many bit strings of length n contain exactly r 1s?
Solution:
The positions of r 1s in a bit string of length n form an r-combination of the set (1, 2, 3, …., n). Hence, there are C(n, r) bit strings of length n that contain exactly r 1s.

Now the teacher should divide the class into six groups of 3 or 4 (Multiple Level of Interpersonal). Then the teacher should explain to the groups that he/she will give the groups examples and they have to solve the problem, however they are going to have to determine whether they have to use permutations or combinations.

Examples
1) How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department, if there are nine faculty members of the mathematics department and 11 of the computer science department?
2) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? (Hint: First position the men and then consider possible positions for the women.)
3) In how many different orders can five runners finish a race if no ties are allowed?

The teacher should then go over those problems with the class as a whole and answer in questions they might have. Now the teacher needs to try and incorporate all the other counting methods into today's lesson. The teacher should do so by giving each group one problem from the past three lessons. Then the teacher should have the each group go to the board and teach the class how to solve their problem. The group will first have to decide which counting technique to use to solve their given problem (Bloom's Level of Evaluation).

Examples
1) How many license plates can be made using either three digits followed by three letters or three letters followed by three digits? (uses the product rule)
2) How many bit strings of length four do not have two consecutive 1s? (uses the tree diagrams)
3) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter. (uses the pigeonhole principle)
4) How many permutations of the letters ABCDEFG contain

A) the string of BCD
B) the string of CFGA
C) the strings BA and GF

5) How many ways are there to seat six people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table? (uses combinations)

When a group is done with their specific problem they should check in with the teacher to make sure it is correct, then put it on the board and wait for everyone else to be done. When everyone is done, then one by one each group should teach the class how to solve their problem and why they used the counting principle that they did.

Closure: The teacher should close by talking real briefly about each one of the counting techniques, and explain when to use the correct ones. The teacher should then have the student restate the formulas that the lesson covered today. Then the students should go back to their seats and make sure everything is ready for the next class. The students should begin their homework until the bell rings.

Adaptations/Enrichments:

When the teacher divides the class into groups there are two options that would be appropriate modifications. The teacher could put all of the GT students into one group and have them working on challenging problems that involve the various counting techniques. This will allow the GT students to stretch their minds and challenge themselves. The other option would be to put one GT student in each group, thus that student could in a sense teach the other members of his/her group. However this could backfire if the GT student is making the other students feel dumb.

If the GT students have already shown mastery over the lesson for today then it would be appropriate to modify their assignment. The book has the easy problems first and the challenging problems later. Thus instead of the odd problems the teacher could say multiples of 4 and have the GT students go up to a higher number. Such as instead of 1-19 odd, you could assign 1-38 multiples of 4. Thus the students will have the same number of homework questions but the GT students will have a much harder assignment.

Also at the beginning of each week you could have the GT students find fun facts about the lessons that you will be teaching that week. GT students are often very passionate about the topic, and will enjoy doing research. While the students are looking for interesting facts, they will also be learning more in depth about the topics. The teacher could meet with the GT students and discuss what they have researched each week and during that time the teacher and students can explore the topics in more detail. Using words such as why, how, always, or never.

Self-Reflection: After the lesson has been taught the teacher should reflect on how successful the lesson was? Are there any ways to improve the lesson? Did the students enjoy the lesson? Did the students learn the standards? Did it work in the time that it was supposed to? Would you teach this lesson again?