Lesson: Probability of Multiple Events
Length: 45 minutes
Age/Grade Intended: Algebra II

Academic Standard(s):
A2.8.2 – Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

Performance Objectives:
Students will determine the probability of simple events involving independent and dependent events with 85% accuracy when given a worksheet containing 25 problems.

Assessment:
Students will be given a worksheet with 25 problems. They will be graded on work shown, how accurately they set up their probabilities, and correctness. It will be possible to receive partial credit for some problems. Point values for each problem are given on the worksheet. The worksheet will be due at the beginning of class the next day.

Advanced Preparation by Teacher:
Two standard decks of 52 playing cards
Probability of Multiple Events Worksheet

Procedure:
• Intro: Use the decks of cards to demonstrate the difference in probabilities of single events (from previous lesson) and probabilities of multiple events.
  o Single events:
    ▪ P(♥) (1/4)
    ▪ P(Jack) (1/13)
    ▪ P(5♣) (1/52)
  o Make conjectures about the following events:
    ▪ P(3♥ then Q♣ from the same deck)?
    ▪ P(A♦ from first deck & a ♠ from the second deck)?
    ▪ P(8 & K)?
- P(7 or Red)?
- Explain that these are examples of multiple events because the probability of more than one event is being taken into account. In other words, more than one thing is happening.
- To find the probability of two events occurring together, you have to decide whether one event occurring affects the other.

➤ Have the students get out their Math Journals and note taking materials.
➤ Classifying Events
  - **Dependent events**
    - Def.: The occurrence of one event affects how a second event can occur.
    - Eqn.: None
    - Pic.: None
    - Ex.: Pick one flash card, then another from a stack of 30 flash cards.
  - **Independent events**
    - Def.: The occurrence of one event does not affect the probability of another event.
    - AND, ×
    - Ex.: Roll a number cube. Then spin a spinner.
    - Pic.: None
    - Eqn.: \( P(A \text{ and } B) = P(A) \cdot P(B) \)

➤ Finding the Probability of Independent Events
  - **Example** – In the cafeteria, there are 10 chocolate milk cartons and 5 regular mild cartons. There are also 8 packages of chocolate chip cookies and 12 packages of peanut butter cookies. If you grab a milk and package of cookies without looking, what is the probability you get regular milk and chocolate chip cookies?
    - Event A = picking regular milk; Event B = picking chocolate chip cookies
    - A and B are independent.
    - \( P(A \text{ and } B) = P(A) \cdot P(B) = \frac{5}{15} + \frac{8}{20} = \frac{11}{15} \)
    - \( P \approx 73\% \)
➤ Mutually Exclusive Events and Finding Probability
  - **Mutually exclusive events**
    - Def.: Two events that cannot happen at the same time. OR, +
    - Ex.: When rolling a dice, you roll a 2 and a 3.
    - Pic. Venn Diagrams can be useful when finding the probability of mutually exclusive events.
    - Eqn.: \( P(A \text{ or } B) = P(A) + P(B) \)
      - (If A and B are independent, then \( P(A \text{ or } B) = P(A) + P(B) \))
  - **Example** – Students at a high school can take one foreign language each term. About 37% take Spanish, and about 15% take French. What is the probability that a student chosen at random is taking Spanish or French?
    - \( P(\text{Spanish or French}) = P(\text{Spanish}) + P(\text{French}) \)
Finding Probability

- If two events are not mutually exclusive: \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \).

- **Example** - The numbers 1 through 10 are written on index cards and placed in a box. What is the probability that a card chosen at random has a number greater than 7 or even?
  \[
  P(>7 \text{ or even}) = P(>7) + P(\text{even}) - P(>7 \text{ and even})
  \]
  \[
  = \frac{3}{10} + \frac{5}{10} - \frac{2}{10} = \frac{6}{10} = \frac{3}{5}
  \]

- Go back to the Intro to check the predictions made at the beginning of the lesson.

  - **P**(drawing one card then Q♣ from the same deck)?
    - Dependent Events – depends on the first draw
  
  - **P**(A♦ from first deck & a ♠ from the second deck)?
    - Independent Events: \( \left(\frac{1}{52}\right) \cdot \left(\frac{1}{4}\right) = \frac{1}{208} \)
  
  - **P**(8 or K)?
    - Mutually Exclusive Events: \( \frac{1}{13} + \frac{1}{13} = \frac{2}{13} \)
  
  - **P**(7 or Red)
    - Not mutually exclusive: \( \frac{1}{13} + \frac{1}{2} - \frac{2}{13} = \frac{11}{26} \)

Adaptations:

- Instead of giving the equations for finding independent event probability and mutually exclusive probability, start by providing examples, and then have the students construct the equations.

- Have the students create a “How To” plan for finding the probabilities of each kind of events.

- Create a worksheet with more challenging problems.

It is crucial for teachers to know their students as learners. Not only is it important to take into consideration the different needs students with disabilities may have, it is also important to consider the needs of any gifted and talented students you may have when writing lesson and unit plans. Just because these students might understand the material you are teaching the class does not mean that we are meeting their needs as learners. These students still need to be engaged and challenged by the material.

In this particular unit on probability and statistics, there are a variety of enrichment activities and ways to differentiate the material to benefit the needs of gifted and talented students. For example, when assigning homework, especially from the first few sections that deal with simpler concepts such as probability of simple events and counting techniques, I can chose questions that require a higher order of thinking. I could challenge them by asking them why things, such as the equations for permutations and combinations, work. I could also have
them derive these formulas. Since basic computing problems might bore these types of learners, it would be more beneficial to limit these computing problems and provide them with more challenging questions like the ones previously mentioned.

There are numerous ways to show students how to apply probability and statistics to real world careers and problems. One type of enrichment activity that would go along with probability is showing students how probability is used in other fields. More specifically, I could show them how a biologist uses Punnett Squares and probability to predict possible gene combinations. When we discuss statistics, I could have them conduct their own survey or experiment and analyze the data they collect by choosing the appropriate methods described in class. Having them analyze their data and methods they chose to use would challenge the students to find meaning in their results.
Probability of Multiple Events

#1-4 Classify each pair of events as dependent or independent. (1pt each)

1. A month is selected at random; a number from 1 to 30 is selected at random.
2. A month is selected at random; a day of that month is selected at random.
3. A letter of the alphabet is selected at random; one of the remaining letters is selected at random.
4. The color of a car is selected at random; the type of transmission is selected at random.

#5-8 A and B are independent events. Find P(A and B). (1pt each)

5. P(A) = 1/6, P(B) = 2/5
6. P(A) = 9/20, P(B) = ¾
7. P(A) = .6, P(B) = .9
8. P(A) = 1/3, P(B) = 6/7

#9-12, C and D are mutually exclusive events. Find P(C or D). (1pt each)

9. P(C) = 2/5, P(D) = 3/5
10. P(C) = .5, P(D) = .375
11. P(C) = 12%, P(D) =27%
12. P(C) = 5/8, P(D) = 1/8
13. A and B are not mutually exclusive. If P(A) = .5, P(B) = .25, and P(A and B) = .125, find P(A or B). (1pt)
14. The sum is a prime number; the sum is less than 4.

15. The numbers are equal; the sum is odd.

16. The product is greater than 20; the product is a multiple of 3.

17. P(3 or odd)
18. P(4 or even)

19. P(even or less than 4)
20. P(odd or greater than 2)

21. P(odd or prime)
22. P(4 or less than 6)

23. You select a blue marble and then a red marble.

24. You select a red marble and then a blue marble.

25. Both of the marbles you select are red.
1. Independent
2. Dependent
3. Dependent
4. Independent
5. 1/15
6. 27/80
7. .54
8. 2/7
9. 1
10. 7/8
11. 39%
12. 3/4
13. 5/8
14. Not mutually exclusive
15. Mutually exclusive
16. Not mutually exclusive
17. 1/2
18. 1/2
19. 5/6
20. 5/6
21. 2/3
22. 5/6
23. 4/15
24. 4/15
25. 1/15

Total points: 43