Logic
Analyzing Extended Arguments
Four Basic Patterns

• Horizontal

• Vertical

• Conjoint Premises

• Multiple Conclusions
(1) The selling of human organs should be outlawed. (2) If this practice is allowed to get a foothold, people in desperate financial straits will start selling their own organs to pay their bills. (3) The criminally-minded will take to killing healthy people and selling their organs on the black market. (4) In the end, this is too much like buying and selling life itself.
(1) The selling of human organs, such as hearts, kidneys, and corneas, should be outlawed. (2) Allowing human organs to be sold will inevitably lead to a situation in which only the rich will be able to afford transplants. This is so because (3) whenever something scarce is bought and sold as a commodity, the price always goes up. (4) The law of supply and demand requires it.
Extended Argument: conjoint premises

(1) Socrates is a Greek and (2) all Greeks are mortal. Therefore (3) Socrates is mortal.
Extended Argument: multiple conclusions

(1) Day traders buy stocks in the morning and sell them at night. As a result, (2) they contribute nothing to the economy, and (3) they also make the markets more volatile.
Practice
Campaign Reform

(1) Campaign reform is needed because (2) many contributions to political campaigns are morally equivalent to bribes.
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(1) The contamination of underground aquifers represents a pollution problem of catastrophic proportions. (2) Half the nation’s drinking water comes from these aquifers and (3) they are being poisoned by chemical wastes dumped into the soil for generations.
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(1) We should not build more nuclear power plants in the United States. (2) Nuclear power is a dangerous technology to those presently living, (3) it places an unfair burden on future generations, and (4) we don’t really need the additional power such plants would generate.
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Composites

(1) The development of carbon-embedded plastics, otherwise called ‘composites’, is an important new technology because (2) it holds the key for new aircraft and spacecraft designs. This is so because (3) these composites are not only stronger than steel but lighter than aluminum.
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(1) Accidents at nuclear power plants are all but inevitable and (2) accidents of this sort generally involve catastrophic consequences. (3) Nuclear power is an unacceptably dangerous technology.
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(1) A worldwide ban on the sale of ivory is long overdue. (2) Without it, the African elephant will become virtually extinct by the year 2000. (3) Today, poachers armed with AK-47 automatic rifles kill 2000 elephants every week, and (4) only 600,000 remain in the wild.
(1) A worldwide ban on the sale of ivory is long overdue. (2) Without it, the African elephant will become virtually extinct by the year 2000. (3) Today, poachers armed with AK-47 automatic rifles kill 2000 elephants every week, and (4) only 600,000 remain in the wild.
Deductive Logic
Overview

1. Distinguishing Deductive and Inductive Logic
2. Validity and Soundness
3. A Few Practice Deductive Arguments
4. Testing for Invalidity
5. Practice Exercises
Deductive and Inductive Logic
Deductive vs Inductive

**Deductive Reasoning**

- Formal (the inference can be assessed from the form alone).
- When sound, the conclusion is *guaranteed* to be true.
- The conclusion is *extracted from* the premises.

**Inductive Reasoning**

- Informal (the inference *cannot* be assessed by the form alone).
- When cogent, the conclusion is only probably true.
- The conclusion *projects beyond* the premises.
Validity

• A property of the form of the argument.

• If an argument is valid, then the truth of the premises guarantees the truth of the conclusion.

Soundness

• A property of the entire argument.

• If an argument is sound, then:
  (1) it is valid, and
  (2) all of its premises are true.
Validity

If an argument is valid, then the truth of the premises guarantees the truth of the conclusion.

A valid argument can have:
• True premises, true conclusion
• False premises, true conclusion
• False premises, false conclusion

A valid argument can not have:
• True premises, false conclusion
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All dogs are mammals.
Ed is a dog.
∴ Ed is a mammal.
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All cats are dogs.
Ed is a cat.
∴ Ed is a dog.
Validity

If an argument is valid, then the truth of the premises guarantees the truth of the conclusion.

A valid argument can have:
- True premises, true conclusion
- False premises, true conclusion
- False premises, false conclusion

A valid argument can not have:
- True premises, false conclusion

All cats are toads.
Ed is a cat.
∴ Ed is a toad.
Sample Deductive Arguments
Deductive Argument #1

(1) If it’s raining, then you’ll need your umbrella.
(2) It’s not raining.
∴ (3) You won’t need your umbrella.
Checking for Invalidity

Two Methods of Counter-example

Alternate scenario

Imagine some alternate scenario in which the premises of the argument will be true, but the conclusion false.

Substitution (two-step)

(1) Determine the form of the argument.
(2) Substitute other statements, such that all the premises will be true but the conclusion false.
Deductive Argument #1

(1) If it’s raining, then you’ll need your umbrella.
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Deductive Argument #1

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s not raining.

∴ (3) You won’t need your umbrella.

(1) If $R$, then $U$  \hspace{1cm} R = \text{I’m a dog.}$

(2) Not-$R$ \hspace{1cm} U = \text{I’m a mammal.}$

∴ (3) Not-$U$

[Denying the Antecedent]

INVALID
Deductive Argument #2

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s raining.

∴ (3) You’ll need your umbrella.
Deductive Argument #2

(1) If it’s raining, then you’ll need your umbrella.

(2) It’s raining.

∴ (3) You’ll need your umbrella.

(1) If R, then U
(1) If P, then Q
(2) R
(2) P
∴ (3) U
∴ (3) Q

[Modus Ponens (Latin: “mode that affirms”)]

VALID
Deductive Argument #3

If Ed has black hair, then Ed is Italian. Ed does have black hair, so Ed is Italian.
Deductive Argument #3

If Ed has black hair, then Ed is Italian.
Ed does have black hair, so Ed is Italian.

(1) If B, then I
(2) B
∴ (3) I

[Modus Ponens]
VALID
Deductive Argument #4

If God exists, then there’s no evil in the world. But there is evil in the world, so God must not exist.
Deductive Argument #4

If God exists, then there’s no evil in the world. But there is evil in the world, so God must not exist.

(1) If G, then not-E
(2) E
∴ (3) not-G

[Modus Tollens (Latin: “mode that denies”)]

VALID
Deductive Argument #5

If the medicine doesn’t work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.
Deductive Argument #5

If the medicine doesn’t work, then the patient will die. The patient did in fact die, so I guess the medicine did not work.

(1) If not-W, then D      If P, then Q
(2)  D                     Q
∴ (3) not-W                 ∴ P

[Affirming the Consequent]
INVALID
Deductive Argument #6

That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.
Deductive Argument #6

That bicycle belongs to either John or Mary. But it looks too big for John. So it must belong to Mary.

(1) J or M
(2) not-J
∴ (3) M

P or Q
not-P
∴ Q

[Disjunctive Syllogism]  VALID
Practice Arguments
Practice Argument #1

If he was lost, then he would have asked for directions. But he didn’t ask for directions. So he must not be lost.

(1) If L, then D  If P, then Q
(2) not-D      not-Q
∴ (3) not-L    ∴ not-P

[Modus tollens]
VALID
Practice Argument #2

If interest rates drop, then the dollar will weaken against the Euro. Interest rates did drop. Therefore, the dollar will weaken against the Euro.

(1) If I, then D
(2) I
∴ (3) D
[Modus ponens]
VALID

If P, then Q
P
∴ Q
Practice Argument #3

If his light is on, then he’s home. But his light isn’t on, so he’s not home.

(1) If L, then H      If P, then Q
(2) not-L        not-P
∴ (3) not-H    ∴ not-Q
[Denying the Antecedent]
INVALID
Practice Argument #4

The mind is an immaterial substance, for it is either identical to the brain or it is an immaterial substance, and it’s not identical to the brain.

(1) B or I  
P or Q
(2) not-B  
not-Q
∴ (3) I  
∴ P
[Disjunctive Syllogism]
VALID
Practice Argument #5

If you want to get into law school, then you’d better do your logic homework.

(1) If L, then H     If P, then Q
[(2) L]       P
[∴(3) H]      ∴ Q

[Enthymeme, expanded as modus ponens]
VALID
Practice Argument #6

If you’re wealthy, then you’ve spent years and years in school. Think about it: If you’re a brain surgeon, then you’re wealthy. And if you’re a brain surgeon, then you’ve spent years and years in school.

(1) If BS, then W
(2) If BS, then S
∴ (3) If W, then S

If P, then Q
If P, then R
∴ If Q, then R

[fallacy]
INVALID
Determining Validity

To determine invalidity…

… we can use the method of counter-example.

To determine validity…

… we need something else: Truth Tables
### Truth Tables

#### Example

1. If I win the lottery, then I’ll buy you dinner.  
   \[ \text{If } p, \text{ then } q \]
2. I won the lottery.  
   \[ p \]
3. I’ll buy you dinner.  
   \[ q \]

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1. Why do conditionals have these truth-values?

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indicates validity
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why do conditionals have these truth-values?
Example

1. If it’s raining, then you’ll need your umbrella. \( \text{If } p, \text{ then } q \)  
2. It’s not raining. \( \text{not}-p \)  
3. You don’t need your umbrella. \( \therefore \text{not- } q \)

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\( P_1 \), \( P_2 \), \( C \): indicates invalidity
The TV of Conditionals

The logic of conditional statements is such that they are false only when the antecedent is true and the consequent is false.

\[ A = \text{If I win the lottery, then I’ll buy you dinner.} \]

Suppose…
(1) I both win the lottery and buy you dinner. \((A \text{ is true})\)
(2) I win the lottery, but don’t buy you dinner. \((A \text{ is false})\)
(3) I lose the lottery, but still buy you dinner. \((A \text{ is true})\)
(4) I lose the lottery, and don’t buy you dinner. \((A \text{ is true})\)
“Or”

In **English**, ‘or’ can be used either inclusively or exclusively:

**Inclusive “or”: “P or Q or both”**
Example: “He’s either reading a book or out in the garden (or both).”

**Exclusive “or”: “P or Q but not both”**
Example: “The train’s coming in on either platform 3 or platform 5.”

In **logic**, “or” is always understood in the inclusive sense.